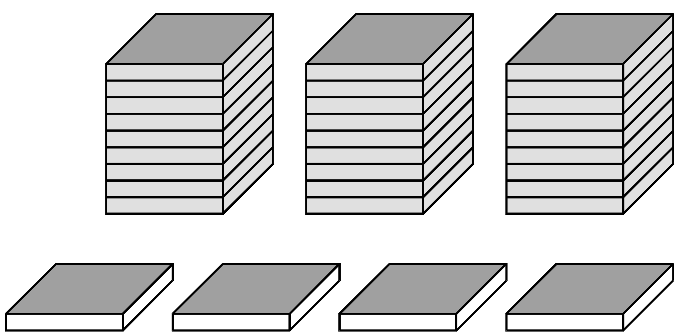
Chapter 7

Equations



Section 1
Introduction to Equations

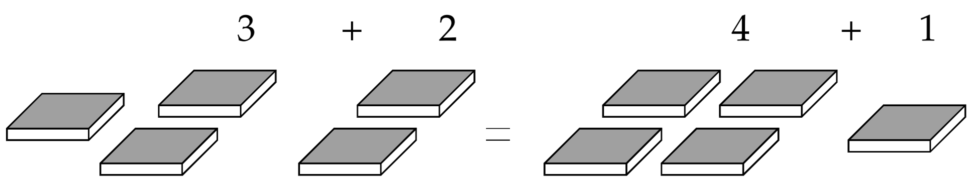
Equations

An **equation** is a number statement which says that two quantities are exactly the same. The symbol **=** (equals) is used between the quantities to show that the amount on the left is the same as the amount on the right. For example:

$$3 + 2 = 4 + 1.$$

Both the numbers on the left and the numbers on the right can be combined to give 5. So the equation really says

$$5 = 5 \quad \text{or} \quad \text{“Five equals Five”}$$

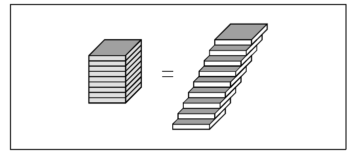


This is obvious if we know all of the numbers on both sides of the equation, but what if the equation has unknowns? When unknowns are included, we can use the fact that both sides are equal to find out the missing amount.

Equations versus Expressions

In the previous chapter, we worked with expressions that involved unknowns. *An expression is a quantity, while an equation is a statement that two quantities are equal.*

Expressions	Equations
$3x + 6$	$x + 3 = 5$
$^{-}17$	$2x - 3 = 15$
$3(5x - 4) + 17x$	$3(5x - 4) + 17x = 20$



An expression can stand for many different amounts, depending on what we choose for the unknown; in an equation, the unknown can only stand for numbers that make both sides of the equation equal. Here is a summary of the differences between equations and expressions:

	Expressions	**Equations**
Quantities	One quantity	Two amounts that are equal
Equals Sign	No	Yes
Meaning of Unknown	Many choices	Values that make the statement true

Exercises

Decide whether each item is an expression or an equation:

1. $x + 3$

2. $2(x - 5) + 7$

3. $0 = 0$

4. $2(x - 5) = 2$

5. $2(x - 5) = 2(x - 5)$

6. $\frac{1}{6}$

7. $\frac{3x}{5} = 12$

8. $3(x + 5) - 16x + 23$

9. ${}^{-}1$

10. 0

11. $0 = 0$

12. $3x + 2 = {}^{-}1$

13. $\frac{3x + 12}{x + 16}$

14. $x = y$

15. $y = 1$

Section 2
The Equation Game

Introduction

This game will help you to understand the meaning of equations and the methods by which they can be solved. As in many of the other sections of this book, you may find that you can easily discover the techniques of solving equations; in fact, you may already know a great deal about the subject.

A sample of this game was presented in the INTRODUCTION. We will now give more detailed rules and examples.

The Rules of the Game

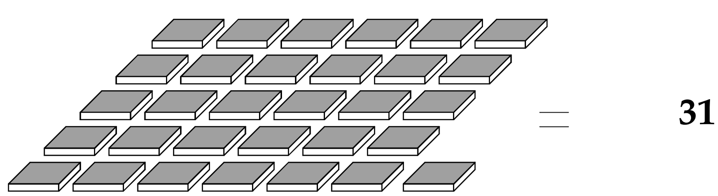
The game can be played alone or with a partner. You can pose equations and solve them yourself, or you can set up equations for your partner to solve.

- **Begin by counting out any number of chips and writing the total in large numerals on one-half of a clean sheet of paper. A number between 10 and 50 chips works best.**

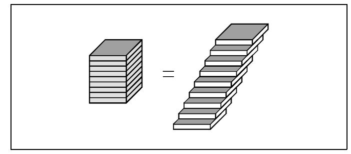
- **Divide up most (but usually not all) of the chips into a small number of stacks which are exactly equal in height.** *If you are playing alone, do not count the number of chips in a stack; you can tell if the stacks are equal by feeling the height of the chips.* **Place these stacks on the other half of the paper with the remaining chips arranged singly next to the stacks.**

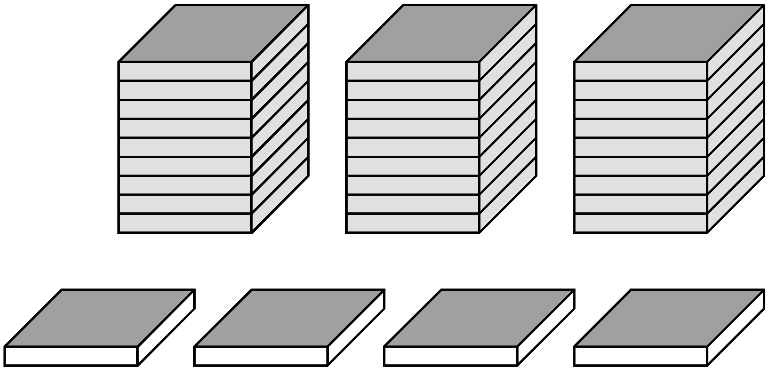
- **Without counting, determine how many chips are in a stack. We know that the total number of chips (stacks and single chips) equals the number written on the paper; use this information to discover how many chips are in a stack. Check the result by counting chips in the stack. If you are not correct, check that the stacks were the same height and that the total number of chips is correct.**

For example, count out 31 chips and write **31** on the paper.



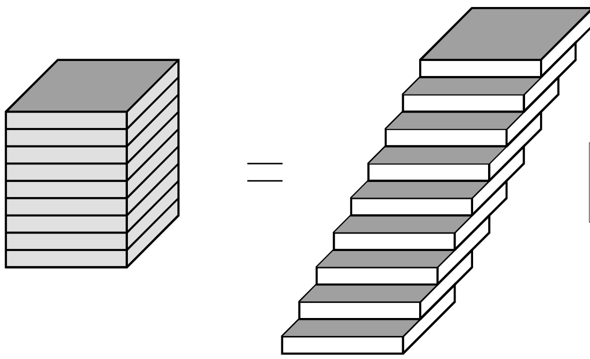
Lay out 4 chips singly and arrange the other chips into 3 equal stacks.





= 31

Calculate the number of chips in each stack. Thirty-one (31) chips minus the 4 extra gives 27 chips, and 27 divided into 3 equal stacks is 9. Check your answer by counting the chips in a stack.

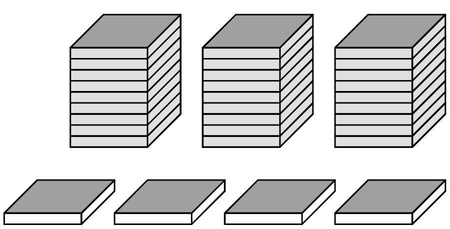


$x = 9$

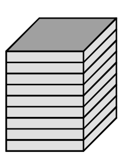
This process is called **solving an equation**. We write the equation as

$$3x + 4 = 31$$

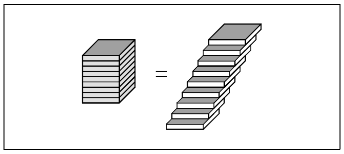
where $\boldsymbol{x}$ is a stack, $\boldsymbol{3x}$ is 3 stacks, and **4** is the 4 single chips.



= 31



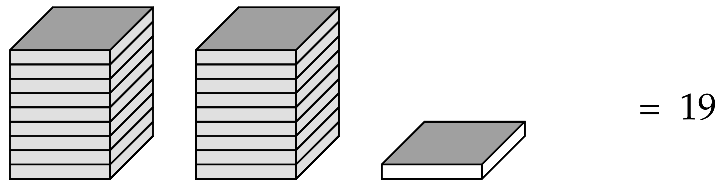
= 9



Exercises

Here are some sample equation games to play. Set up the chips, calculate the solution, and check your answer by counting the chips in a stack.

Example: $2x + 1 = 19$
Solution: $x = 9$

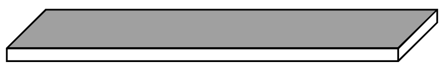


1. 23 chips: 4 stacks and 3 singles. ($4x + 3 = 23$)
2. 17 chips: 2 stacks and 1 single. ($2x + 1 = 17$)
3. 35 chips: 4 stacks and 3 singles.
4. 12 chips: 1 stack and 5 singles.
5. 29 chips:
   (You arrange stacks and singles. All stacks are the same height.)
6. 32 chips: 3 stacks and 11 singles.
7. 23 chips: 2 stacks and 5 singles.
8. 27 chips: 4 stacks and 7 singles.
9. 27 chips: 6 stacks and 3 singles.
10. 27 chips: 4 stacks and 3 singles.
11. 18 chips: 3 stacks and 3 singles.
12. 41 chips: 3 stacks and 5 singles.
13. 32 chips: 4 stacks and 4 singles.
14. 51 chips: 5 stacks and 6 singles.
15. 40 chips: 3 stacks and 1 single.
16. 47 chips: 2 stacks and 5 singles.
17. 38 chips: 5 stacks and 3 singles.
18. 50 chips: 6 stacks and 2 singles.
19. 35 chips: 4 stacks and 3 singles.
20. 29 chips: 3 stacks and 2 singles.

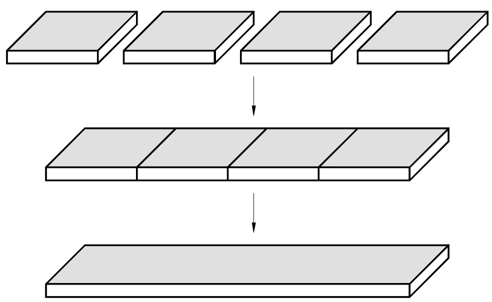
Section 3
Equations Using Unknowns

Using the Bar as *x*

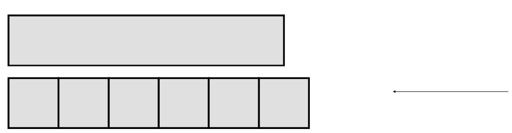
As we learned in the last chapter, we can also represent an unknown amount with the long bar found in your packet.



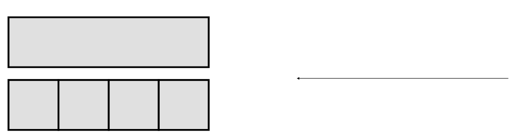
Instead of a stack of chips in a pile, the bar represents a group of chips in a line:



The bar *represents any unknown number of chips; you can imagine that it changes length in each example:*

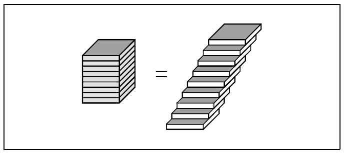


Actual Proportions: The bar is 5 ½ units long.



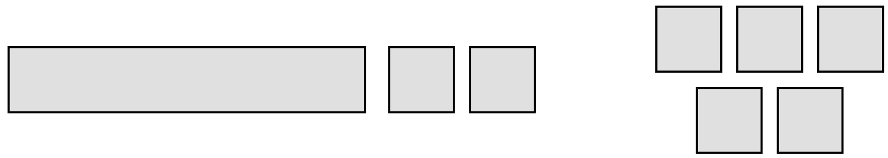
But it may stand for a quantity of 4 units.

Or it may stand for 2.



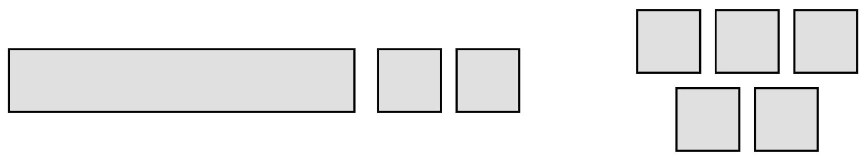
Equations Using Bars

In the following equation, what number does the bar represent?

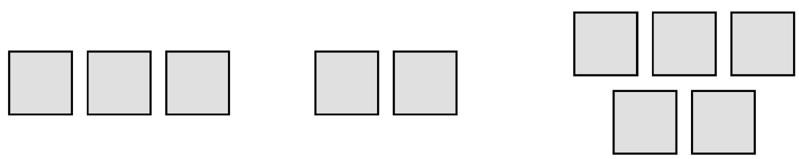


$x \quad + \quad 2 \quad = \quad 5$

We are trying to find out what number of chips are needed to replace the bar so that both sides of the equation are equal. The answer is 3, so the bar represents three chips:



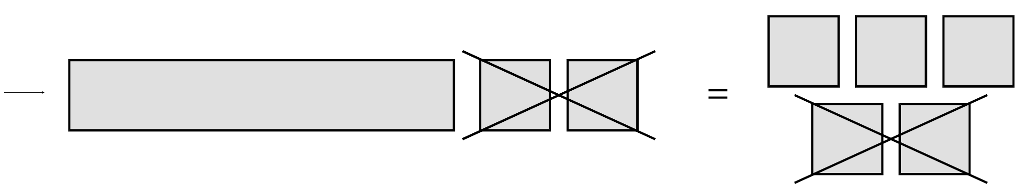
$x \quad + \quad 2 \quad = \quad 5$



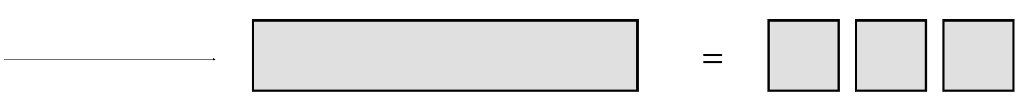
$3 \quad + \quad 2 \quad = \quad 5$

The best way to find the answer is to take chips away from each side until one side has only the bar left. In this case, we take 2 chips away and the bar must then be equal to 3 chips.

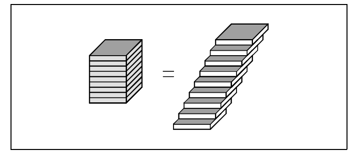
Take 2 chips away from each side.



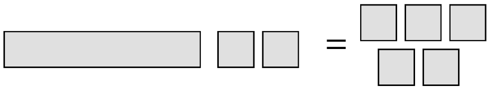
This leaves the bar equal to 3.



To check, replace the bar with 3 chips and make sure that there are equal numbers of chips on both sides.

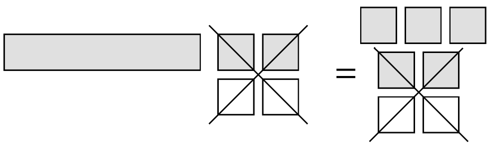


Another way to solve this equation is to add 2 negative chips to each side. Here is the process along with the algebra symbols that we will use:



$$x + 2 = 5$$

Represent the equation using a bar (unknown) and unit chips.



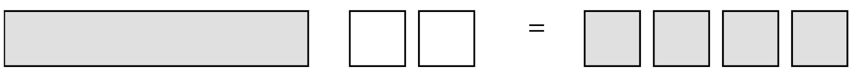
$$x + 2 + {}^{-}2 = 5 + {}^{-}2$$

Add ${}^{-}2$ to both sides.

$$x = 3$$

This leaves the solution: the value for the unknown.

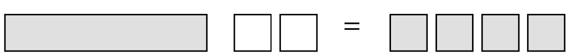
Here is a slightly different example:



$$y + ({}^{-}2) = 4$$

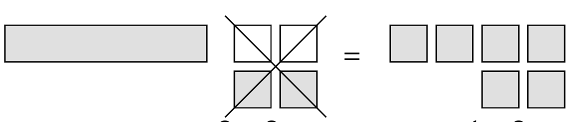
In order to find the value of the unknown (called ***y*** for variety) we look for a number that, when combined with ${}^{-}2$, becomes 4. The answer is ${}^{+}6$.

To solve this more easily, we can work to isolate the *y* bar by *adding* 2 positive chips to each side. This will cancel the negative chips and will help us discover the answer:



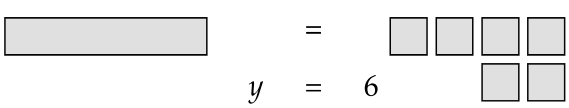
$$y - 2 = 4$$

Represent the equation using a bar (unknown) and unit chips.



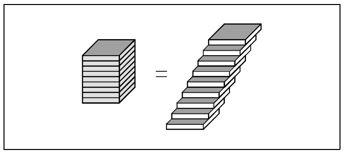
$$y - 2 + 2 = 4 + 2$$

Add ${}^{+}2$ to both sides.



$$y = 6$$

This leaves the solution: $y = 6$



Remember that

$$6 + {}^{-}2 = 6 - 2$$

You can use either form, but with chips it is often easier to represent the idea of adding $^{-}2$.

To find the solution:

- **"Isolate" the unknown by adding unit chips to both sides so that the chips other than the bar are cancelled out.**
- **Use positive chips to cancel negative chips, and negative chips to cancel positive chips.**
- **You are done when you have the bar alone, equal to a number of chips.**

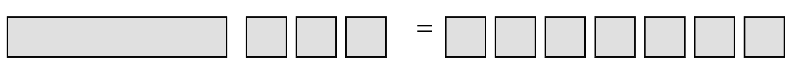
To cancel out units:

Add the Opposite

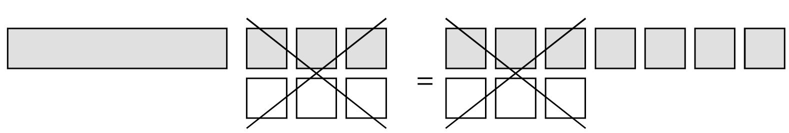
Exercises

Practice on these examples. Use the chips and also write out the algebra symbols for each problem.

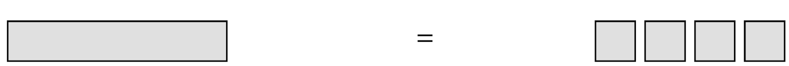
Example: $x + 3 = 7$
Solution:



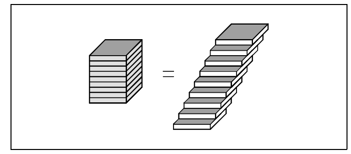
$$x + 3 = 7$$



$$x + 3 - 3 = 7 - 3$$



$$x = 4$$

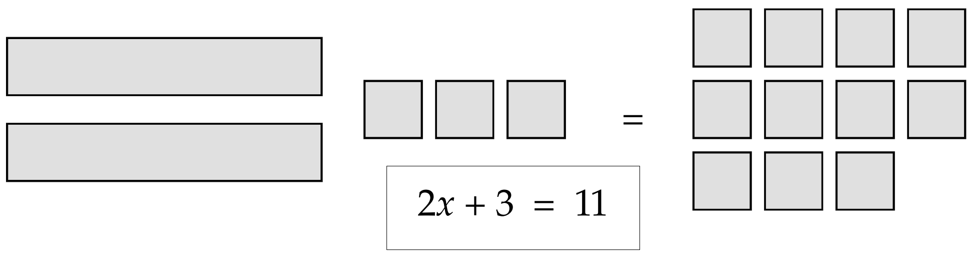


1. $x + {}^-4 = 5$
2. $x + {}^-2 = {}^-3$
3. $y + 5 = 2$
4. $n - 4 = {}^-1$
5. $y + 2 = 2$
6. $x - 7 = 5$
7. $11 + x = 12$
8. $3 + y = {}^-13$
9. $1 + y = 0$
10. $x - 12 = 11$
11. $x + 12 = {}^-3$
12. $y - ({}^-3) = 5$
13. $y + ({}^-3) = 5$
14. ${}^-2 + x = 13$
15. $x + 0 = 0$
16. $x - 5 = 12$
17. $x + 5 = 12$
18. $y - 2 = {}^-3$
19. $y + 2 = {}^-3$
20. $5 + x = {}^-2$
21. $7 + y = 4$
22. $n + 6 = 5$
23. $n - 3 = 5$
24. $x - 7 = {}^-7$
25. $x + 7 = {}^-7$

Section 4
Equations with Multiples of Unknowns

More than One Unknown

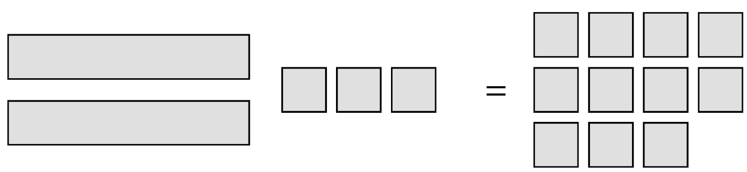
An equation may be more complicated than those that we have looked at thus far. For example, an equation may contain more than one x:



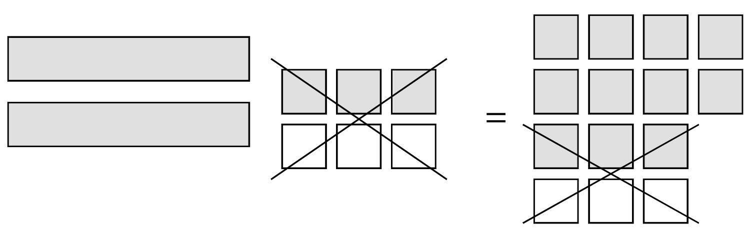
$2x + 3 = 11$

The first step in solving the equation is the same as for the simpler equations—we add $^{-}3$ to each side to isolate the unknowns. This gives:

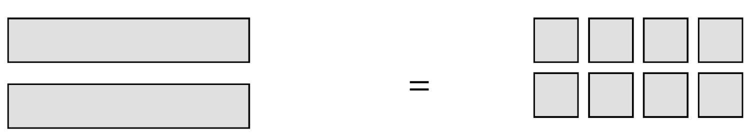
$2x + 3 = 11$

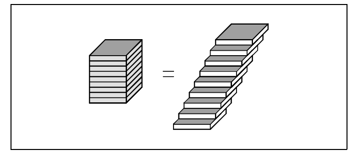


$2x + 3 - 3 = 11 - 3$

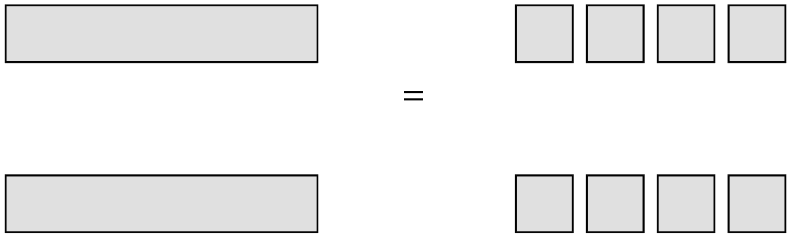


$2x = 8$

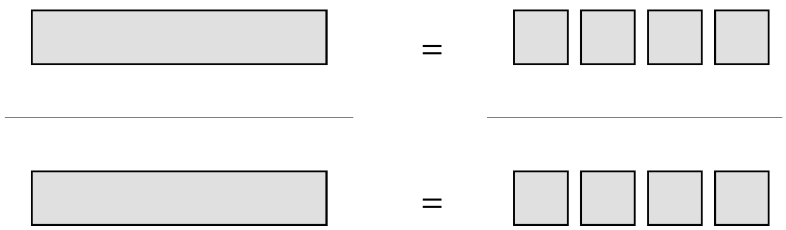




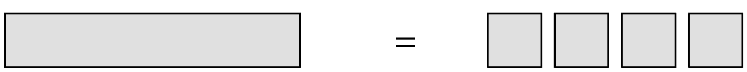
Although we now know what $2x$ is, we would like to know the value of x itself. Because the two sides are equal, we can split up the x bars into 2 groups and the unit chips into 2 groups.



If the two sides are equal, then we can match up half of one side with half of the other side:



$$\frac{1}{2}(2x) = \frac{1}{2}(8)$$

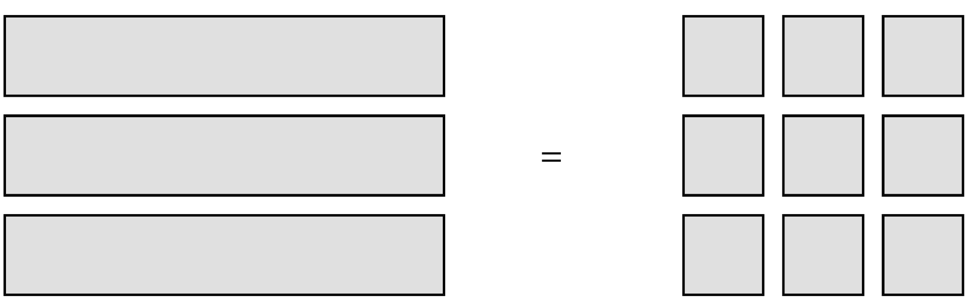


$$x = 4$$

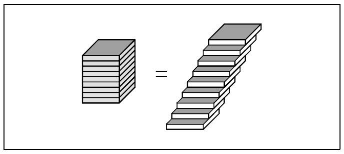
Our solution is 4.

Now we will do another example. Consider

$$3x = 9$$

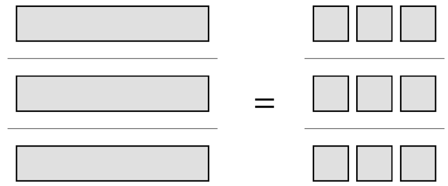


$$3x = 9$$

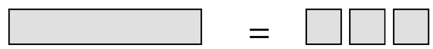


We divide each side into 3 groups and match up one group on each side giving x equal to 3:

$$\frac{1}{3}(3x) = \frac{1}{3}(9)$$

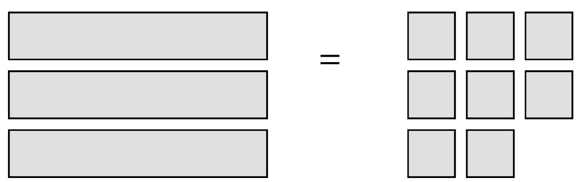


$$x = 3$$



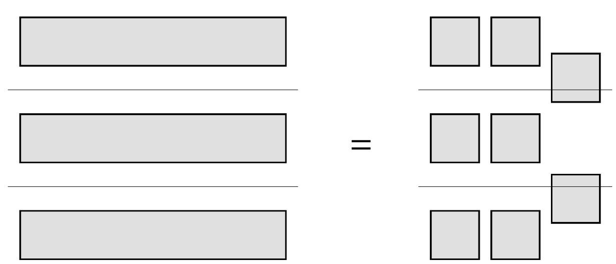
Equations that Result in Fractions

Sometimes an equation will result in a situation where the chips cannot be divided evenly into the desired number of groups. In these cases, the answer will contain a fraction. For example, $3x = 8$:

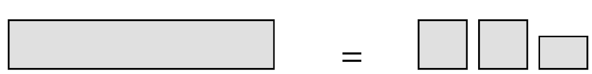


When we divide both sides into thirds, some chips on the right side must be cut to get 3 equal parts. The result is that $x = 2\,{}^{2}\!/_{3}$:

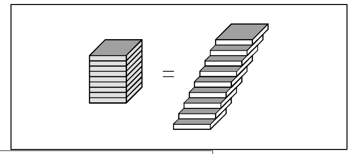
$$\frac{1}{3}(3x) = \frac{1}{3}(8)$$



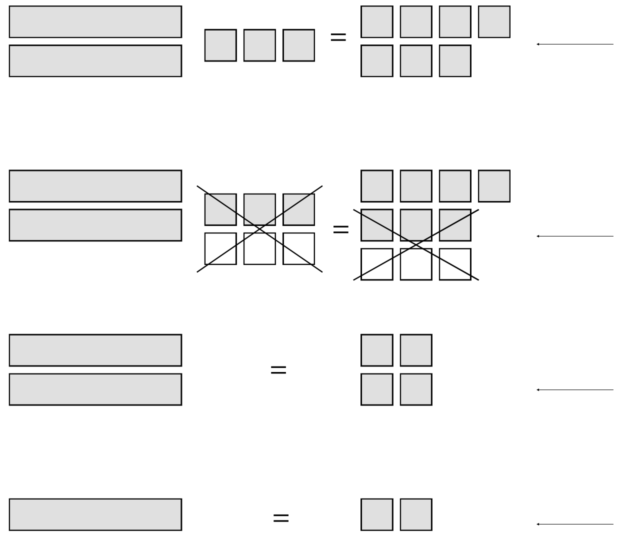
$$x = 2\frac{2}{3} = \frac{8}{3}$$



Exercises



Use chips to solve these equations. Write out the algebra steps for



$2x + 3 = 7$

$2x + 3 - 3 = 7 - 3$

$2x = 4$

$x = 2$

each problem:

Example: $2x + 3 = 7$

Solution:

1. $3x + 5 = 17$

2. $3x + 4 = {}^{-}17$

3. $4x - 3 = 5$

4. $5x + 2 = 11$

5. $2y - 9 = {}^{-}5$

6. $6n - 2 = 3$

7. $2b + 5 = 5$

8. $5x + 1 = 11$

9. $2 + 3x = 35$

10. $3x - 2 = 8$

11. $6 + 3y = 21$

12. $0 + 2x = 0$

13. ${}^{-}3 + 5x = {}^{-}13$

14. ${}^{-}2 + 4x = {}^{-}10$

15. $3y - 12 = 12$

16. $7n + 7 = 8$

17. $3x + 1 = 5$

18. $5x = 3$

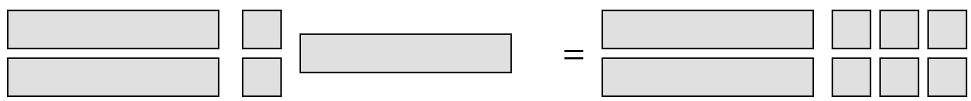
19. $2x - 1 = 0$

Section 5
Unknowns in More than One Term

Keeping the Equation Balanced

An equation may have unknowns in several places—on one side of the equation or on both sides. Consider the following equation:

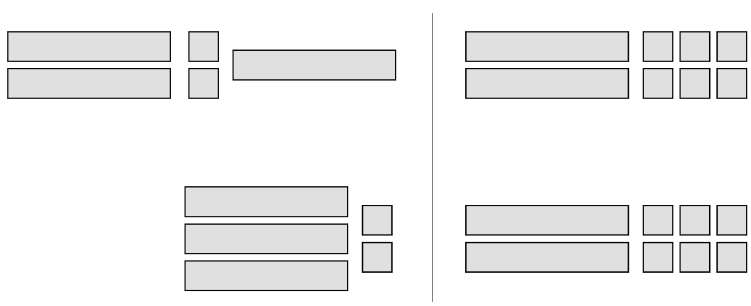
$$2x + 2 + x = 2x + 6$$



$$2x + 2 + x = 2x + 6$$

In an equation like this it is important to notice the position of the equals sign because it separates the left and right sides. The equation is like a balance and the amount on the left exactly balances the amount on the right.

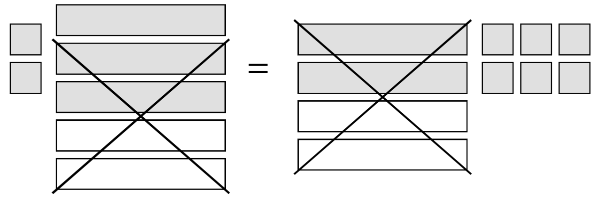
Our first step is to combine like terms on each side:



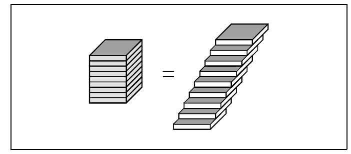
$$3x + 2 = 2x + 6$$

The next step is to add or remove unknowns from both sides. *We must add or remove equally on both sides or the equations will not remain balanced.* Since the right side has less unknowns, we can cancel out these by adding two negative bars to both sides:

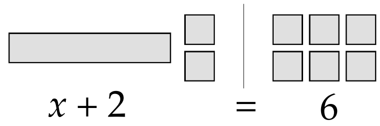
$3x - 2x + 2$



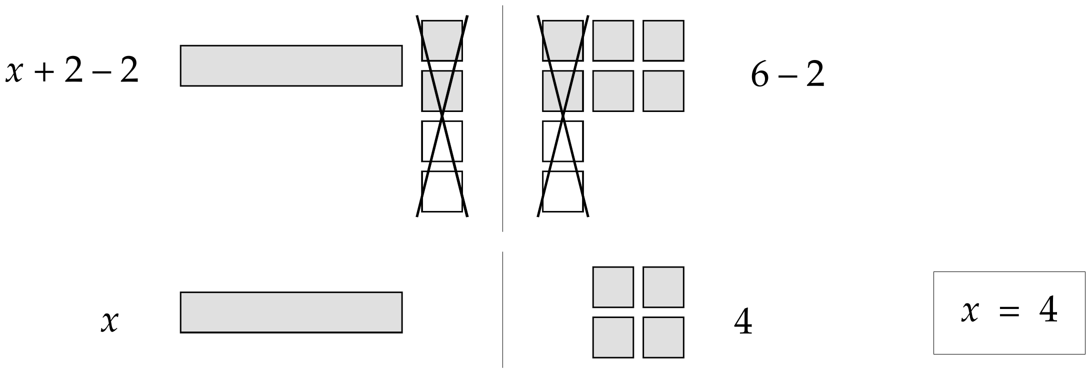
$2x - 2x + 6$



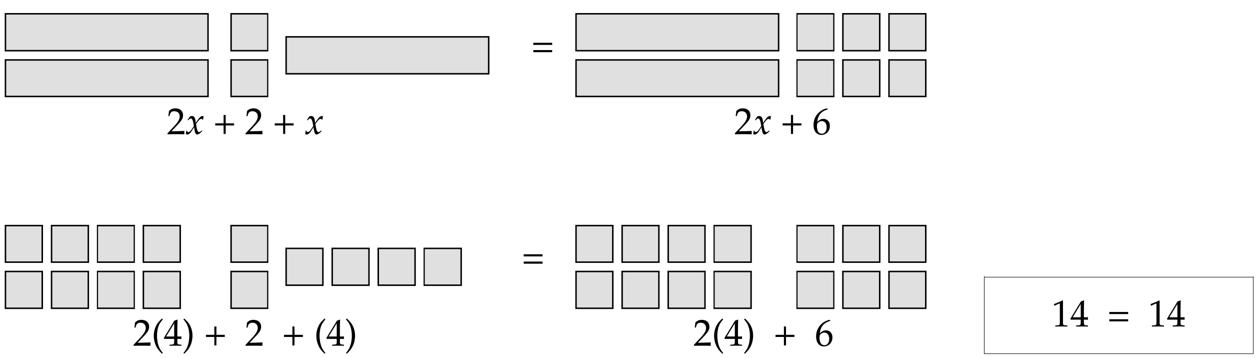
This leaves us with unknowns on one side only. When we combine similar terms, we are left with:



From this point, we can solve the equation exactly as before:

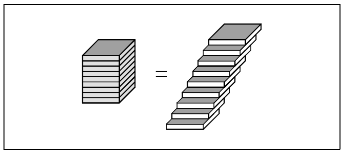


Our solution is that $x = 4$. To check our answer, we replace each x bar on both sides of the equation with 4 chips and then confirm that both sides have a balanced (equal) number of chips:

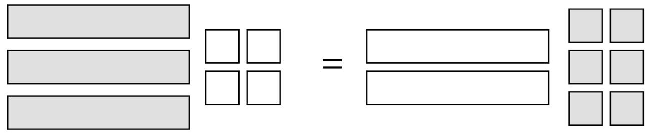


To summarize these steps:

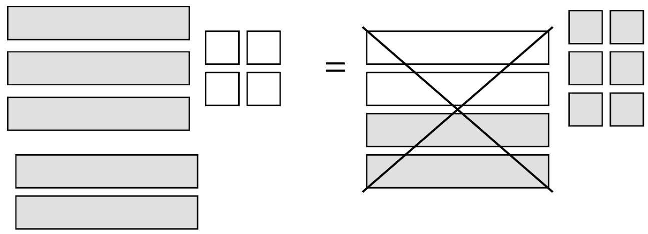
- **Combine similar terms on each side of the equation.**
- **Eliminate the unknowns from one side by adding the opposite type of bars. Add negatives to eliminate positives, and add positives to eliminate negatives.**
- **Add positive or negative chips to cancel out the units and to "isolate" the unknown.**
- **Multiply both sides by ½, ⅓, etc. to match up a single unknown with the correct number of chips.**



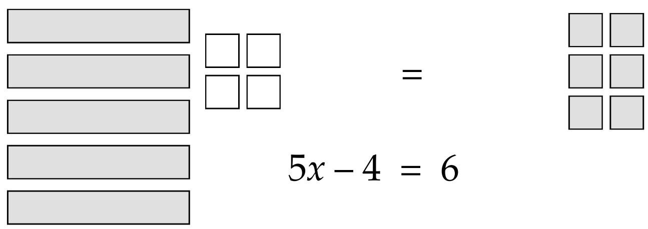
Here is another example:



$$3x - 4 = {}^{-}2x + 6$$

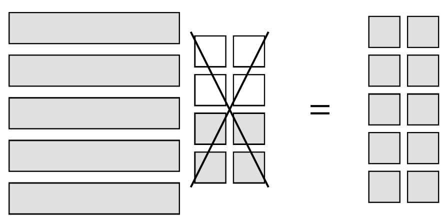


$$3x + 2x - 4 = {}^{-}2x + 2x + 6$$

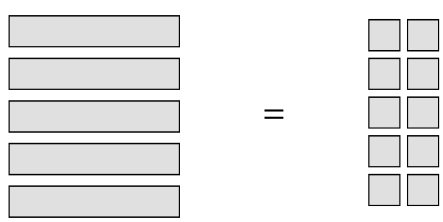


$$5x - 4 = 6$$

We can now solve as before:

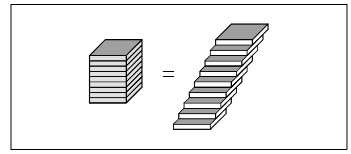


$$5x - 4 + 4 = 6 + 4$$



$$5x = 10$$

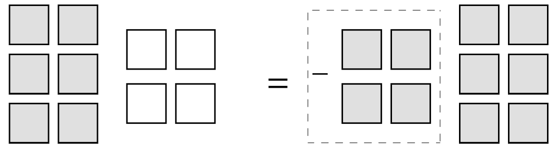
$$x = 2$$



To check our result of $x = 2$, we replace the x's with 2 chips and the $-x$'s with $^{-}2$ chips:

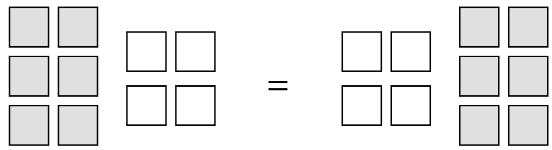
$$\begin{aligned} 3x - 4 &= {}^{-}2x + 6 \\ 3(2) - 4 &= {}^{-}2(2) + 6 \\ 6 - 4 &= {}^{-}4 + 6 \\ 2 &= 2 \end{aligned}$$

$3(2) - 4$



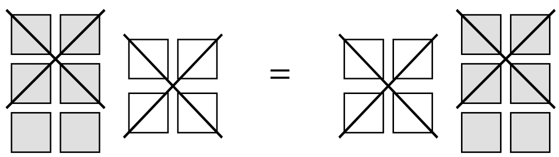
$^{-}2(2) + 6$

$6 - 4$



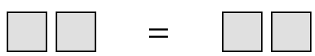
$^{-}4 + 6$

$6 - 4$



$^{-}4 + 6$

2

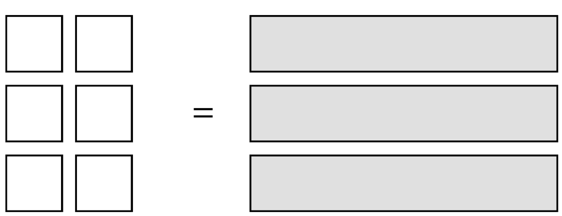


2

$$2 = 2$$

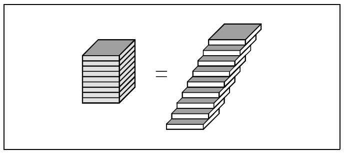
Unknowns on the Right Side of the Equation

When we isolate the unknowns, the unknowns may be on the right side of the equation instead of on the left.



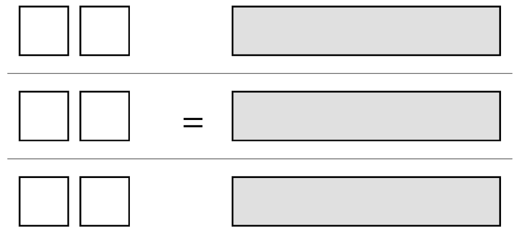
$$^{-}6 = 3x$$

Because

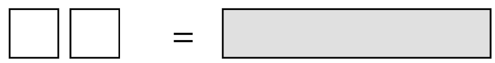


$$^{-}6 = 3x \quad \text{has the same meaning as} \quad 3x = {}^{-}6$$

we do not have to swap the sides of the equation. Instead, we continue solving in the usual way:



$$\frac{1}{3}(^{-}6) = \frac{1}{3}(3x)$$



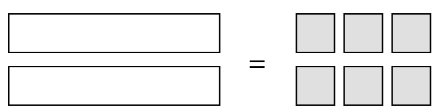
$$^{-}2 = x$$

Negative Unknowns

In the final step of solving an equation, we may be left with negative unknowns:

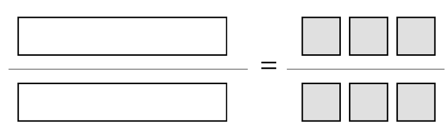
We can use our usual method to isolate the negative x by multiplying both sides by one-half:

$$^{-}2x = 6$$

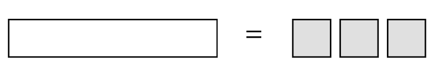


But now we have the value of the *opposite* of x instead of the value of x itself. It is clear that if the opposite of x is 3, then x is $^{-}3$. We can show this physically

$$\frac{1}{2}(^{-}2x) = \frac{1}{2}(6)$$

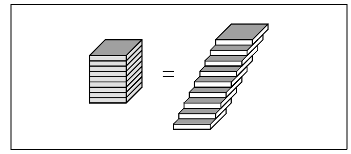


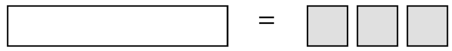
$$-x = 3$$



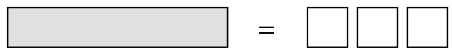
by flipping all of the chips on both sides:

This keeps our equation balanced, because *if two quantities are equal, their opposites are also equal.* With symbols, it is often written as:





$-x = 3$



$x = {}^{-}3$

$$\begin{aligned} -x &= 3 \\ {}^{-}1(-x) &= {}^{-}1(3) \\ x &= {}^{-}3 \end{aligned}$$

Multiplying both sides by negative one can be shown as flipping the chips on both sides.

Exercises

Do these problems using the chips. Write out the steps.

1. $3x + 5 - x = x - 6$

2. $2x - 4 + x = -x + 8$

3. ${}^{-}2y - 2 + y = 2y + 7$

4. $6 - 3n = n + 5 - 3$

5. $4y - 3 = {}^{-}3 - 6y$

6. $1x + 2x + 3x = 4x + 4$

7. $4 - z - 2z - 3z = {}^{-}20$

8. $6x = 2x - 12$

9. $7x - 5x + x = 14 + x$

10. $10 + x = {}^{-}12x + 5x - 6$

11. $9y + 2 = 6y - 4$

12. $-x = 5$

13. $-x = {}^{-}3$

14. ${}^{-}7x = {}^{-}3x$

15. ${}^{-}2x + 6 = x - 9$

16. $-x + 6 = -2x + 2$

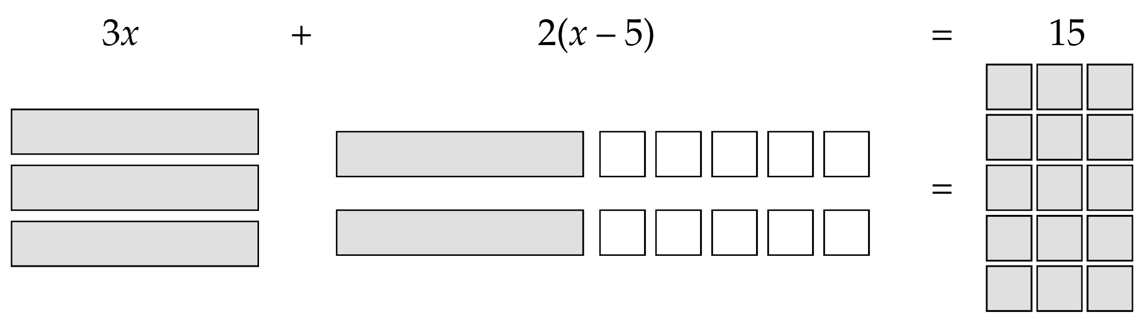
17. $-x + 1 = 5x + 1$

Section 6
Equations with Parentheses

Using the Distributive Property

Some equations may contain complicated expressions including parentheses. For example:

$$3x + 2(x - 5) = 15$$



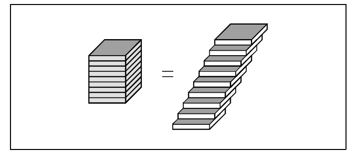
Notice that the 2 is not a term. It cannot be eliminated by adding $^{-}2$ to both sides. In most cases, it is necessary to use the distributive property to multiply out the expression **2(*x* – 5)** so that we can combine terms and proceed with the solution of the equation:

$$\begin{aligned} 3x + (2 \cdot x) - (2 \cdot 5) &= 15 \\ 3x + 2x - 10 &= 15 \\ 5x - 10 &= 15 \\ 5x &= 25 \\ x &= 5 \end{aligned}$$

Consider the equation:

$$5 + 3(x + 2) = 2(x + 1) + 12$$

Again, we cannot eliminate any of the parts of $3(x + 2)$ or $2(x + 1)$ until we multiply out these expressions using the distributive property:



$$\begin{aligned} 5 + 3(x + 2) &= 2(x + 1) + 12 \\ 5 + 3x + (3\cdot2) &= (2 \cdot x) + (2\cdot1) + 12 \\ 5 + 3x + 6 &= 2x + 2 + 12 \end{aligned}$$

From here on, the procedure is the same as in the previous section:

$$\begin{aligned} 5 + 3x + 6 &= 2x + 2 + 12 \\ 11 + 3x &= 2x + 14 \\ 11 + 3x - 11 &= 2x + 14 - 11 \\ 3x &= 2x + 3 \\ 3x - 2x &= 2x + 3 - 2x \\ x &= 3 \end{aligned}$$

Subtraction and the Distributive Property

When the product of two amounts is *subtracted* in an equation, we must be careful to use signed numbers correctly. Consider the equation:

$$5 - 2(x + 1) = 1$$

This is not the same as:

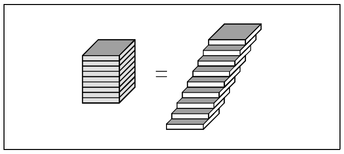
$$5 - 2x + 1 = 1 \quad \text{(Why not?)}$$

Instead, rewrite the subtraction as addition:

$$\begin{aligned} 5 - 2(x + 1) &= 1 \\ 5 + {}^{-}2(x + 1) &= 1 \\ 5 + ({}^{-}2 \cdot x) + ({}^{-}2 \cdot 1) &= 1 \\ 5 + {}^{-}2x + {}^{-}2 &= 1 \\ 3 + {}^{-}2x &= 1 \\ {}^{-}2x &= {}^{-}2 \\ x &= 1 \end{aligned}$$

If there are two subtractions, you may want to rewrite both as addition:

$$\begin{aligned} 10 - 3(2x - 5) &= 19 \\ 10 + {}^{-}3(2x + {}^{-}5) &= 19 \\ 10 + ({}^{-}3 \cdot 2x) + ({}^{-}3 \cdot {}^{-}5) &= 19 \\ 10 + {}^{-}6x + 15 &= 19 \\ {}^{-}6x + 25 &= 19 \\ {}^{-}6x &= {}^{-}6 \\ x &= 1 \end{aligned}$$



Summary

We can now add an initial step to our plan from the previous section:

- ***Use the distributive property to complete any multiplications of expressions in parentheses.***

- **Combine similar terms on each side of the equation.**

- **Eliminate the unknowns from one side by adding the opposite type of bars. Add negatives to eliminate positives, and positives to eliminate negatives.**

- **Add positive or negative chips to cancel out the units and to "isolate" the unknown.**

- **Multiply both sides by ½, ⅓, etc. to match up a single unknown with the correct number of chips.**

Exercises

Solve these equations using chips and algebra symbols:

1. $4(x + 1) - 3 = 3(x - 2) + 13$

2. $5 - 2(x - 2) = 5(x + 1) + 4$

3. $3(2x + 1) - 2(x - 1) = 21$

4. $6(1 - x) + 3 = {}^{-}3x - 3$

5. $3(3 + 2x - 1) = 2x - 1(3 - 2x) + 9$

6. $5(x + 1) - 4x = 2$

7. $2x + 3(x - 2) = 9x + 2$

8. $7y - 6(y - 1) + 3 = 9$

9. $3 - y(1 + 2) = {}^{-}2y - 5$

10. $2(3x + 1) - 5(x + 2) = 1 - 10$

11. $3(7 - 2x) = 14 - 8(x - 1)$

12. $1 - 3(x + 4) = {}^{-}5x - 5$

13. $1 - 1(x - 1) = x$

14. $5x + 6x + 3(x - 2) = {}^{-}6 + x$

15. $3x + 2(3x - 1) = 6x$

16. $5(x - 2) - 3x = 6$

17. $2x - 9 = 3(2 - x)$

18. $5 - 2(1 - x) = 3(x - 4)$

19. $2(4x + 3) - 3(x - 2) = 3x + 8$

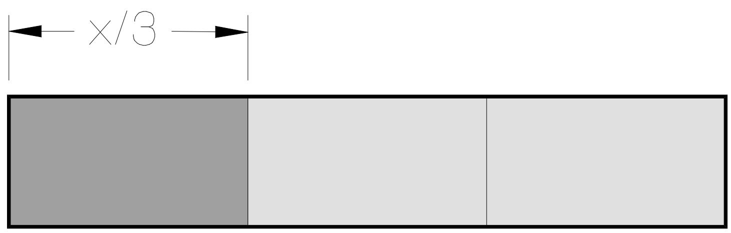
20. $8x - 3(2x + 5) = x - 4$

Section 7
Equations with Fractions or Decimals

Simplifying Equations with Fractions

If fractions occur in an equation, there is an easy technique for creating an equivalent equation without fractions. For example:

$$\frac{x}{3} + 4 = x$$



First, think of $x/3$ as x divided into 3 pieces, or one-third of x.
Although equations of this type can be solved in the usual way by subtracting parts of x from both sides, it is usually easier (with symbols or chips) to multiply both sides of the equation by a number so that the resulting equation has no fractions. *Multiplying both sides of an equation by the same number creates an equivalent equation with the same solution as the original equation.*

$$\frac{x}{3} + 4 = x$$

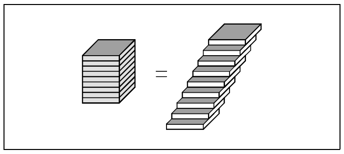
$$3\left(\frac{x}{3} + 4\right) = 3(x)$$

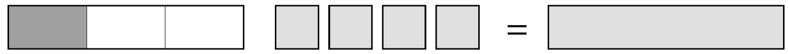
$$3\left(\frac{x}{3}\right) + 3(4) = 3x$$

$$\frac{3x}{3} + 12 = 3x$$

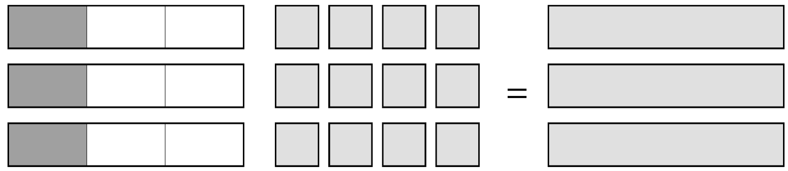
$$x + 12 = 3x$$

Notice that you must multiply 3 times each term on both sides.

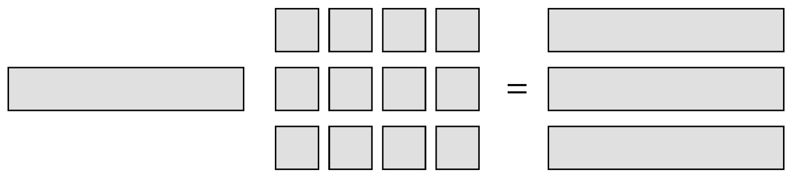




$$\frac{x}{3} + 4 = x$$



$$3\left(\frac{x}{3}\right) + (3)(4) = 3(x)$$



$$x + 12 = 3x$$

After this point, the steps are the same as in previous sections:

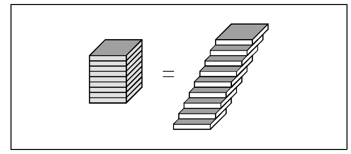
$$\begin{aligned} x + 12 &= 3x \\ x + 12 - x &= 3x - x \\ 12 &= 2x \\ \frac{1}{2}(12) &= \frac{1}{2}(2x) \\ \frac{12}{2} &= \frac{2x}{2} \\ 6 &= x \end{aligned}$$

How do you choose the number that you will use to multiply? If x is divided by 3, multiply by 3. If x is divided by 4, multiply by 4. If we triple $x/3$ (one-third of x), we will get one x.

If the equation has more complicated fractions, we still multiply by a number which will cancel the denominators. For example:

$$\frac{2}{3}x + 2 = \mathrm{x}$$

It is still useful to multiply both sides by a number, and we use the same number as in the previous example:



$$\frac{2}{3}x + 2 = x$$

$$3\left(\frac{2}{3}x + 2\right) = 3(x)$$

$$3\left(\frac{2}{3}x\right) + 3(2) = 3(x)$$

$$\left(\frac{3}{1} \cdot \frac{2}{3}\right)x + 6 = 3x$$

$$\frac{6}{3}x + 6 = 3x$$

$$2x + 6 = 3x$$

$$2x + 6 - 2x = 3x - 2x$$

$$6 = x$$

Equations with Several Fractions

An equation may contain fractions with *different* denominators. We will still multiply both sides by a number, but this time we will use a number that will eliminate all of the fractions.

The number we want will have to be divisible by all of the denominators, or it will not "cancel" when it is multiplied times each term. The lowest number that is divisible by a group of numbers is called the **least common multiple** or **least common denominator**. Here is an example of the steps:

$$\frac{x}{3} + \frac{x}{4} = \frac{x}{2} + 1$$

The least common denominator of 3, 4, and 2 is 12.

$$12\left(\frac{x}{3} + \frac{x}{4}\right) = 12\left(\frac{x}{2} + 1\right)$$

$$12\left(\frac{x}{3}\right) + 12\left(\frac{x}{4}\right) = 12\left(\frac{x}{2}\right) + 12(1)$$

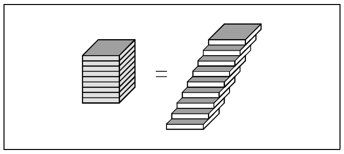
$$\frac{12x}{3} + \frac{12x}{4} = \frac{12x}{2} + 12$$

$$4x + 3x = 6x + 12$$

$$7x = 6x + 12$$

$$7x - 6x = 6x - 6x + 12$$

$$x = 12$$



Equations with Decimals

Decimal numbers such as .1 and 3.034 are often called decimal fractions because they represent fractions with denominators of 10, 100, 1000, etc. Because decimals are really fractions, we solve equations with decimals in the same way that we solve equations with fractions.

Consider this equation:

$$.3x + .2 = 1.7$$

We find a number (the least common multiple) that we can use to multiply times both sides to eliminate the decimals. The correct choice is to multiply by 10, because the equation could be written as:

$$\frac{3}{10}x + \frac{2}{10} = \frac{17}{10}$$

Multiplying by 10 will eliminate the decimals and will result in a new equation that is easier to solve:

$$\begin{aligned} 10(.3x + .2) &= 10(1.7) \\ (10 \cdot .3x) + (10 \cdot .2) &= 17 \\ \mathbf{3x + 2} &= \mathbf{17} \end{aligned}$$

We can now solve the equation in the usual way:

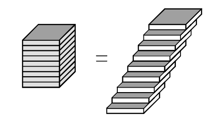
$$\begin{aligned} 3x + 2 &= 17 \\ 3x + 2 - 2 &= 17 - 2 \\ 3x &= 15 \\ x &= 5 \end{aligned}$$

Equations may also contain decimals with different numbers of decimal places. Again, we multiply both sides by the power of 10 (10, 100, 1000, etc.) that will eliminate decimals from all of the numbers. Consider this equation:

$$.03x + .7 = x - 3.18$$

There are three numbers with decimal points. Two of them (.03 and 3.18) have two decimal places and one (.7) has one decimal place. We need to multiply by 100 to eliminate all of the decimal places:

$$\begin{aligned} 100(.03x + .7) &= 100(x - 3.18) \\ 3x + 70 &= 100x - 318 \\ 3x + 388 &= 100x \\ 388 &= 97x \\ 4 &= x \end{aligned}$$



To check our answer:

$$
\begin{aligned}
.03x + .7 &= x - 3.18 \\
.03(4) + .7 &= (4) - 3.18 \\
.12 + .7 &= .82 \\
.82 &= .82
\end{aligned}
$$

To review, we chose 100 as our number to multiply because it was the least common multiple. We could have rewritten the original equation to show why this is true:

$$.03x + .7 = x - 3.18$$

is also:

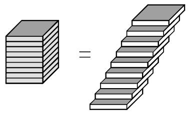
$$\frac{3}{100}x + \frac{7}{10} = x - \frac{318}{100}$$

The common denominator for 10 and 100 is clearly 100.

Fractions and Decimals: How to Multiply

With both fractions and decimals, we multiply both sides of the equation by the least common multiple. With fractions, we look at the denominators and choose the least common denominator. With decimals, we look at the number of decimal places and we multiply by the appropriate power of 10 (10, 100, 1000 …).

For this equation:	Multiply by:	Reason:
$\frac{x}{6} + \frac{x}{8} = 7$	24	Common denominator of 6 and 8
$\frac{x}{3} + \frac{3x}{4} = \frac{x}{6} + 11$	12	Common denominator of 3, 4, and 6
$.02 + .13x = .15$	100	Maximum of 2 decimal places
$3 + .001x = 3.1$	1000	Maximum of 3 decimal places



It is important to understand that you do not have to multiply these equations, but it is usually easier to do so. If you do not multiply both sides, you can solve the equations by subtracting and dividing with the fractions or decimals:

$$
\begin{aligned}
.03x + .7 &= x - 3.18 \\
.03x + .7 + 3.18 &= x - 3.18 + 3.18 \\
.03x + 3.88 &= x \\
.03x + 3.88 - .03\text{x} &= x - .03x \\
3.88 &= .97\text{x} \\
\frac{3.88}{.97} &= \frac{.97}{.97}x \\
4 &= x
\end{aligned}
$$

Summary

Now we can add one more step to our list:

- **Use the distributive property to complete any multiplications of expressions in parentheses.**
- ***If fractions or decimals are present, multiply both sides of the equation by the least common multiple (least common denominator).***
- **Combine similar terms on each side of the equation.**
- **Eliminate the unknowns from one side by adding the opposite type of bars. Add negatives to eliminate positives, and positives to eliminate negatives.**
- **Add positive or negative chips to cancel out the units and to "isolate" the unknown.**
- **Multiply both sides by ½, ⅓, etc. to match up a single unknown with the correct number of chips.**

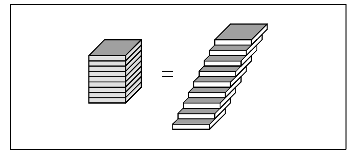
Exercises

Solve for x.

1. $\frac{x}{12} + 1 = x - 21$

2. $\frac{x}{2} = x + 4$

3. $\frac{x}{2} = {}^{-}1$



4. $\frac{x}{2}+\frac{x}{3} = \frac{5}{6}$

5. $\frac{x}{3}+x = 7$

6. $x-\frac{x}{4} = 9$

7. $\frac{x}{3}+\frac{x}{2} = \frac{x}{4}+\frac{7}{2}$

8. $\frac{2x}{3}+2 = \frac{2}{3}$

9. $\frac{1}{5}x+3 = 6$

10. $\frac{3}{4}x+1 = x-2$

11. $3-\frac{2}{3}x = 4x-5$

12. $6.9 + x = 3.3$

13. $.3 + 2x = 3.9$

14. $.2x + 3.1 = 3.9$

15. $.02 + .13x = .15$

16. $3 + .001x = 3.1$

17. $.2x + .8 = x - 4$

18. $3x + 4x = 6.8 + .2x$

19. $.002x = 0$

20. $\frac{3}{5}x+.2 = .1x+10.2$

(What is the common denominator for 5 and 10?)

Section 8
Special Solutions

When the Variable Disappears

Some equations may contain the same number of x's on both sides. This may be obvious:

$$3x + 7 = 3x + 7$$

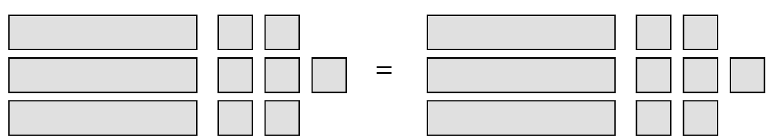
or it may occur after you have combined similar terms:

$$\begin{aligned} 3x + 2(x + 1) &= 5(x + 1) \\ 3x + 2x + 2 &= 5x + 5 \\ 5x + 2 &= 5x + 5 \end{aligned}$$

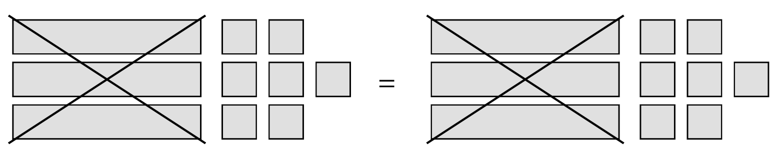
If you proceed in the usual way by subtracting x's, you will get strange results. In the first case:

$$\begin{aligned} 3x + 7 &= 3x + 7 \\ 3x + 7 - 3x &= 3x + 7 - 3x \\ 7 &= 7 \end{aligned}$$

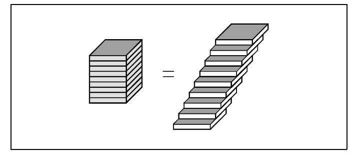
$3x + 7 = 3x + 7$



$3x + 7 - 3x = 3x + 7 - 3x$

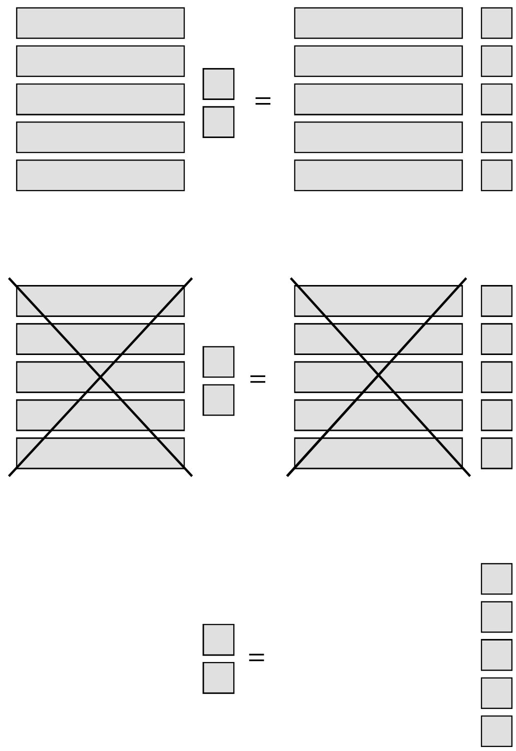


$7 = 7$



In the second case:

$$5x + 2 = 5x + 5$$
$$5x + 2 - 5x = 5x + 5 - 5x$$
$$2 = 5$$



The two cases above have a different meaning:

- **7 = 7 Since this is a true statement, the equation will be true for any x. You can choose any value for x, and the equation is still true. The equation is true for all x.**

- **2 = 5 Since this statement is false, the equation is false for any x. We substitute any value for x, but the equation will always turn out to be false. There is no solution.**

Exercises

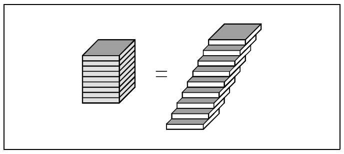
Solve for x. Determine if there is a solution, if there is no solution, or if the equation is true for all values.

1. $2x + 3x = 5x$

2. $4x + 3x = 7x + 1$

3. $2(x + 1) = 8$

4. $2(x + 1) - 2 = 3(x + 4) - (x + 12) + 1$



5. $3(3 + 2x - 1) = 4x - 1(3 - 2x) + 9$

6. $2(x + 3) - 2x = 5$

7. $2x + 3 = 3$

8. $^{-}3(x - 1) = x + 4(2 - x) - 5$

9. $1 + x - (x - 1) = 2$

10. $3(x + 1) = 3x + 1$