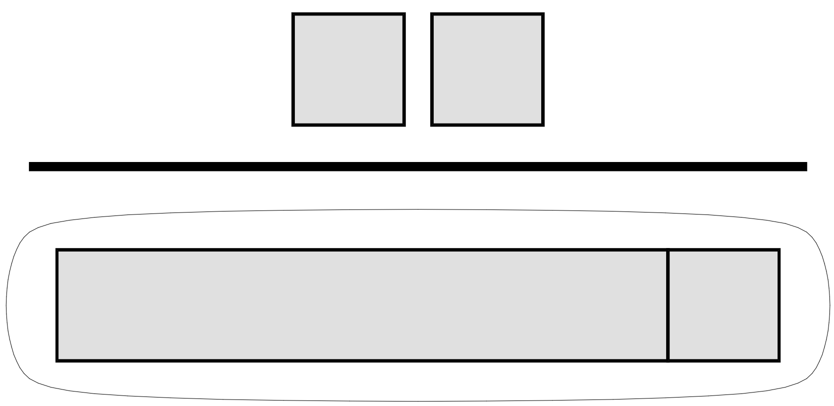
Chapter 14
Rational Expressions



Section 1
Introduction

Rational Expressions

Rational expressions are expressions written as **ratios**. When we write ratios with integers, (numbers such as 1, 2, or 25), we call them **fractions**. The ratio of 1 to 4 is written

$$1:4 \text{ or } \frac{1}{4}$$

Rational expressions are also fractions, except that they can contain both numbers and unknowns. For example:

$$\text{The ratio of } 3 \text{ to } x \text{ is } \frac{3}{x}$$

$$\text{The ratio of } x+5 \text{ to } x^2-7x+2 \text{ is } \frac{x+5}{x^2-7x+2}$$

$$\text{The ratio of } x^2+2x \text{ to } 7 \text{ is } \frac{x^2+2x}{7}$$

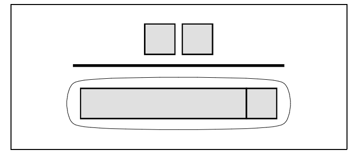
We work with rational expressions in essentially the same way as we work with fractions; in some sections of this chapter we will briefly review common fractions before expanding the discussion to more complex rational expressions.

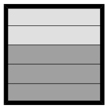
Fractions

You will recall that the fraction $3/5$ can be pictured in several different ways:

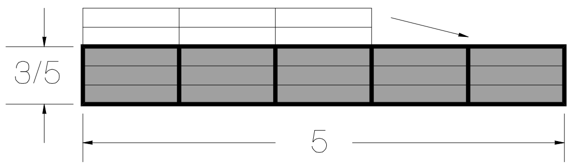
- **$\frac{1}{5}$ of 3**
- **3 parts out of 5**
- **The height of a rectangle made with an area of 3 units arranged so it is 5 units wide.**

$\frac{1}{5}$ of 3



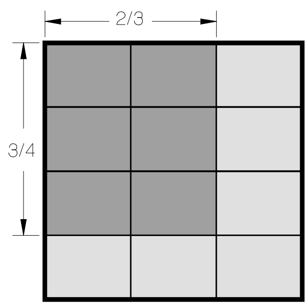


3 parts out of 5



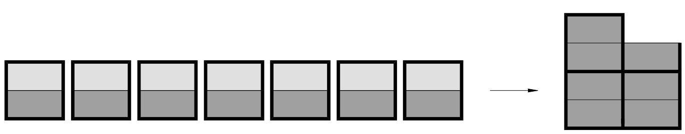
The height of a rectangle made with an area of 3 units, arranged so it is 5 units wide.

We have also represented fractions as smaller rectangles within unit rectangles, such as

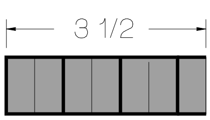


$$\frac{2}{3} \cdot \frac{3}{4} = \frac{6}{12}$$

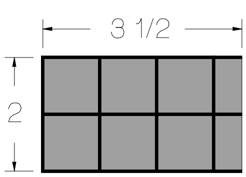
Of course, fractions need not be limited to ratios which are less than one whole unit. The fraction $7/2$ can be illustrated as:



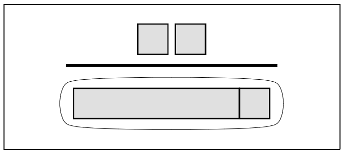
$\frac{1}{2}$ of 7



7 parts where each unit has 2 parts



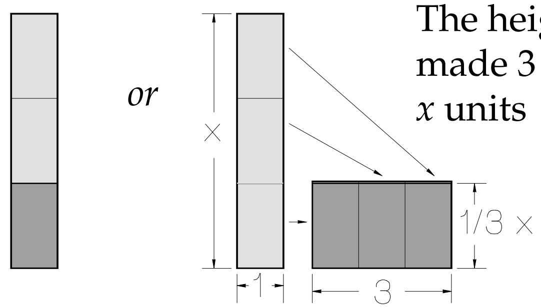
The width of a rectangle made 2 units high using 7 units



Fractions Containing Unknowns

Picturing fractions which contain unknowns is no more difficult than picturing common fractions. For example, $x/3$ can be pictured as

$\frac{1}{3}$ of x

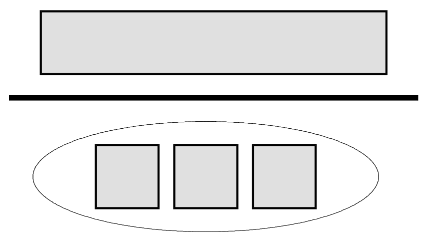


or

The height of a rectangle made 3 units wide using x units

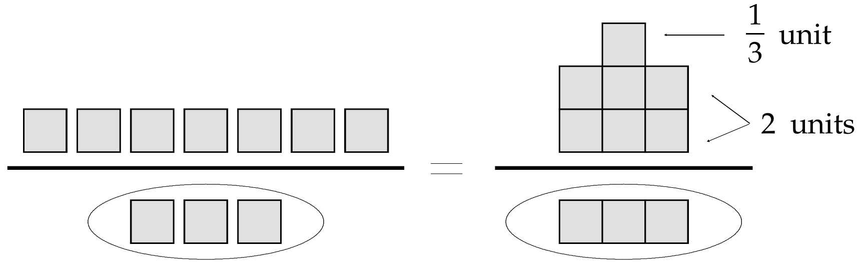
But the third picture, where we show x parts when each unit has 3 parts, is harder to draw. We can suggest a new method for representing this case:

x parts when a unit is 3 parts

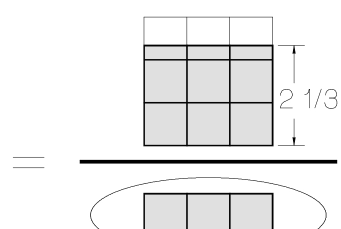


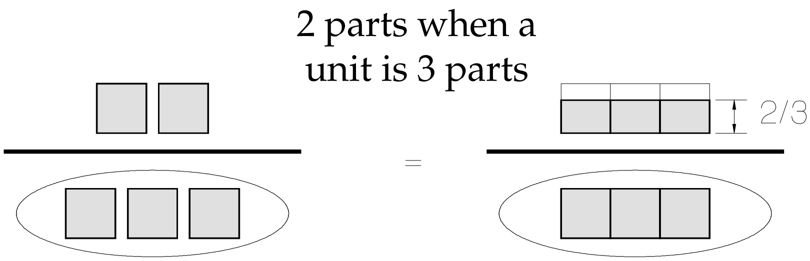
Here, the circled denominator represents the size of one unit. We can see what this representation means by substituting some particular values for the unknown (x), and then grouping them into the units shown in the denominator.

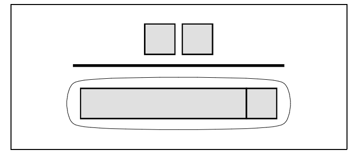
7 parts when a unit is 3 parts

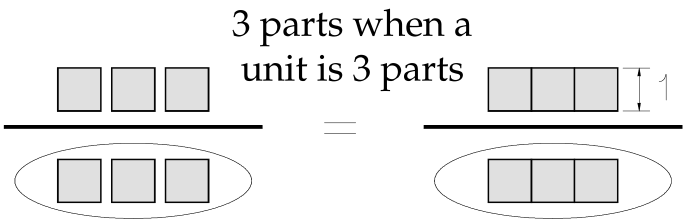


or $2\frac{1}{3}$ groups of 3



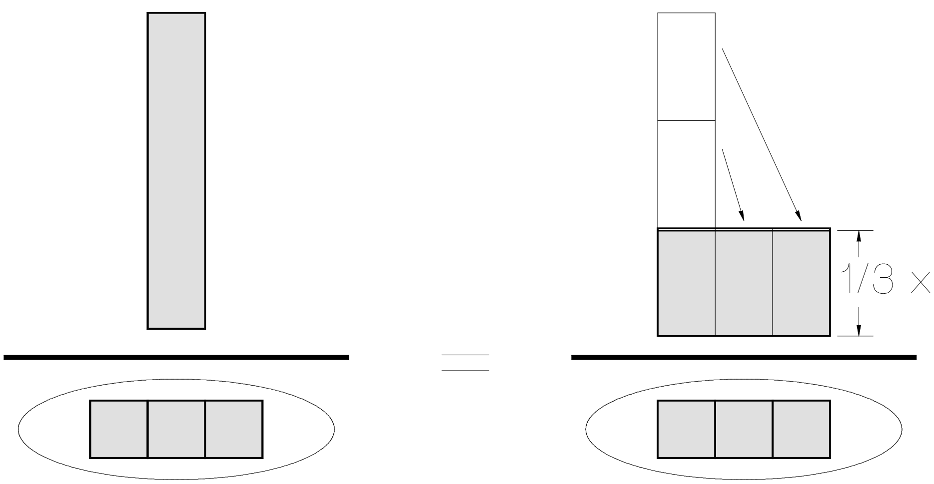






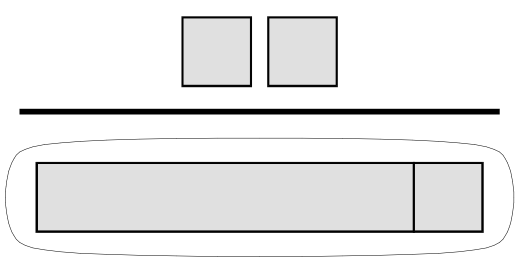
The denominator (bottom pieces) forms a rectangle which stands for one unit; we then use the top pieces to make as many of these unit rectangles as we can, with any left over part (**remainder**) still being written as a fraction.

If we don't know the value of x, we can sometimes represent the fraction using more than one picture, but the value of the result will still be a fraction containing an unknown.



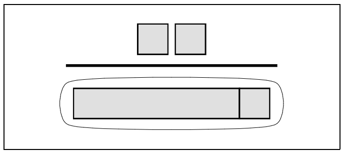
$$\frac{x}{3}$$

When we have unknowns in the denominator, our new representation will still work, but alternative pictures are sometimes harder to visualize:



$$\frac{2}{x+1}$$

This expression represents 2 chips spread out into a rectangle which is $x + 1$ units wide.



Exercises

Draw a picture to represent each of the following rational expressions:

1. $\frac{4}{5}$

2. $\frac{3}{7}$

3. $\frac{5}{4}$

4. $\frac{12}{5}$

5. $\frac{x}{5}$

6. $\frac{x}{2}$

7. $\frac{2x}{3}$

8. $\frac{3x}{2}$

9. $\frac{x+3}{4}$

10. $\frac{x+1}{3}$

11. $\frac{2}{x}$

12. $\frac{3}{x+2}$

Section 2
Simplifying Rational Expressions

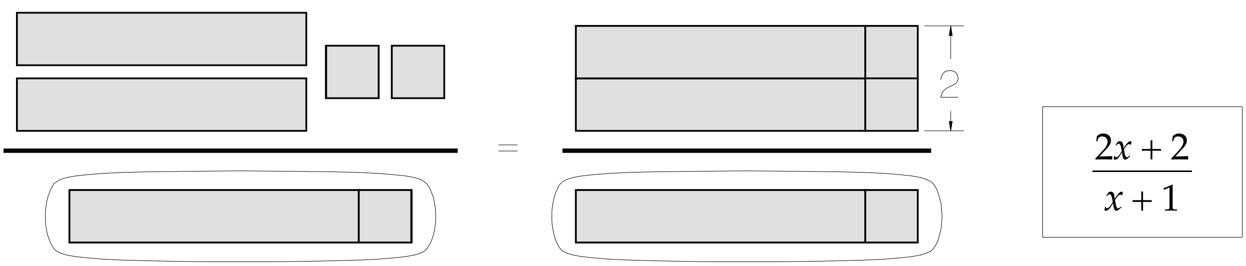
Simplifying

Consider the last expression from the previous section:

$$\frac{2}{x+1}$$

This expression cannot be simplified without knowing the value of x. However, the following expression can easily be simplified:

$$\frac{2x+2}{x+1}$$



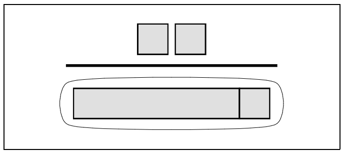
This expression simplifies to 2, because the pieces in the numerator combine to make exactly two unit rectangles (denominators).

Collapsing, Reducing, and Canceling

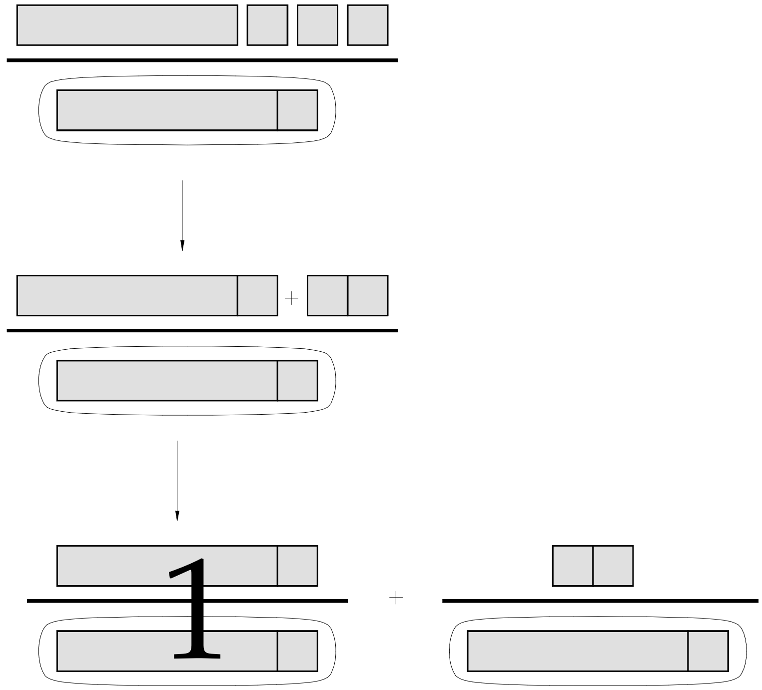
Let's look at some more rational expressions. What if we begin with

$$\frac{x+3}{x+1}$$

What can we do with this expression? Here we separate our original expression into the sum of two fractions: one where the numerator matches the denominator (thus reducing to one), and the second which has the left-over pieces (**remainder**) over the same denominator:



$$\frac{x+3}{x+1}$$



$$\frac{(x+1)+(2)}{x+1}$$

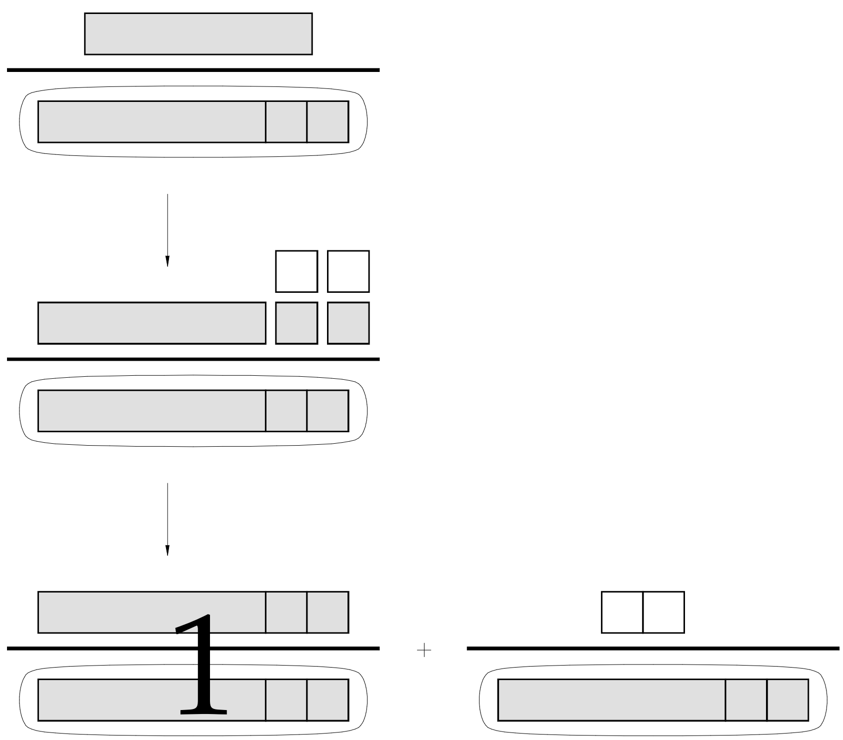
$$\frac{x+1}{x+1}+\frac{2}{x+1} = 1+\frac{2}{x+1}$$

An expression such as

$$\frac{x}{x+2}$$

can be simplified in the following way:

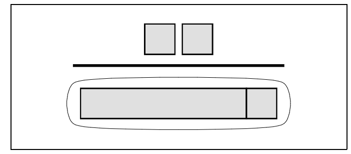
$$\frac{x}{x+2}$$

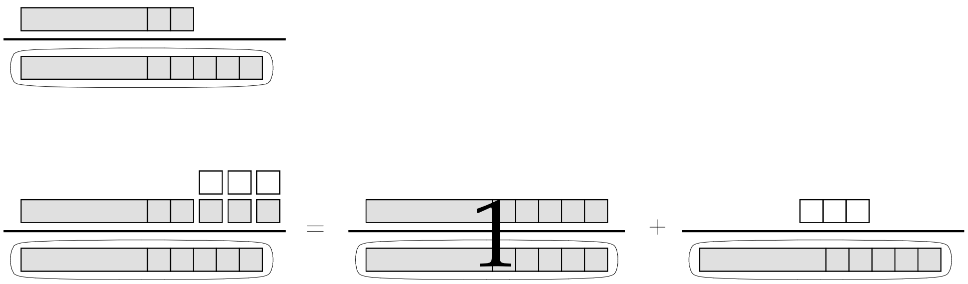


$$\frac{x+2-2}{x+2}$$

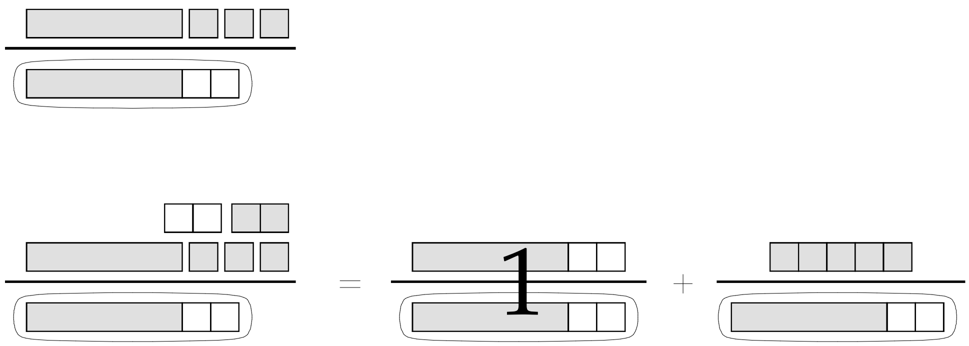
$$\frac{x}{x+2} = 1+\frac{^-2}{x+2}$$

Here we have used the idea of adding equal numbers of positive and negative chips to the numerator (top) so that we can complete one unit. Some further examples of this technique follow:

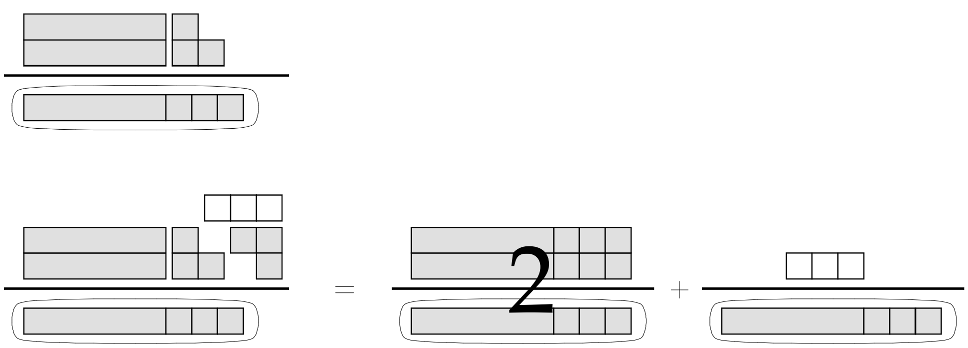




$$\frac{x+2}{x+5} = 1 + \frac{^{-}3}{x+5}$$

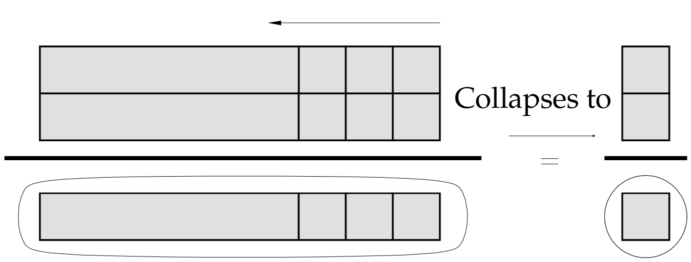


$$\frac{x+3}{x-2} = 1 + \frac{5}{x-2}$$



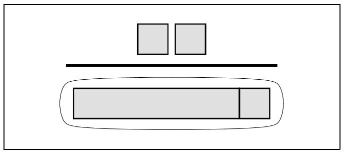
$$\frac{2x+3}{x+3} = 2 + \frac{^{-}3}{x+3}$$

In all of these examples, when the numerator (top) of the fraction matches the denominator (bottom) of the same fraction along one dimension, then the fraction reduces to being just a whole number. This is equivalent to both the top and bottom of the fraction collapsing along their direction of common length. In the symbolic language of algebra, it is the same as dividing out (canceling) like factors from the top and bottom of the fraction.



Cancels to

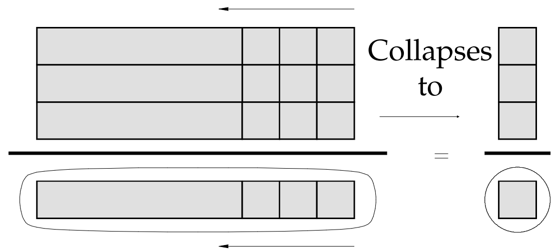
$$\frac{2x+6}{x+3} = \frac{2\cancel{(x+3)}}{1\cancel{(x+3)}} = \frac{2}{1}$$



This only works when one dimension of the top and bottom of the fraction are exactly the same.

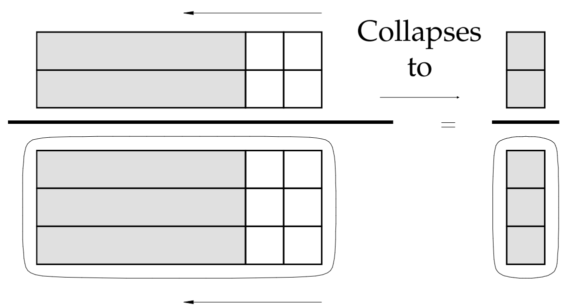
Cancels

$$\frac{3x+9}{x+3} = \frac{3\cancel{(x+3)}}{1\cancel{(x+3)}} = \frac{3}{1}$$



Cancels

$$\frac{2x-4}{3x-6} = \frac{2\cancel{(x-2)}}{3\cancel{(x-2)}} = \frac{2}{3}$$



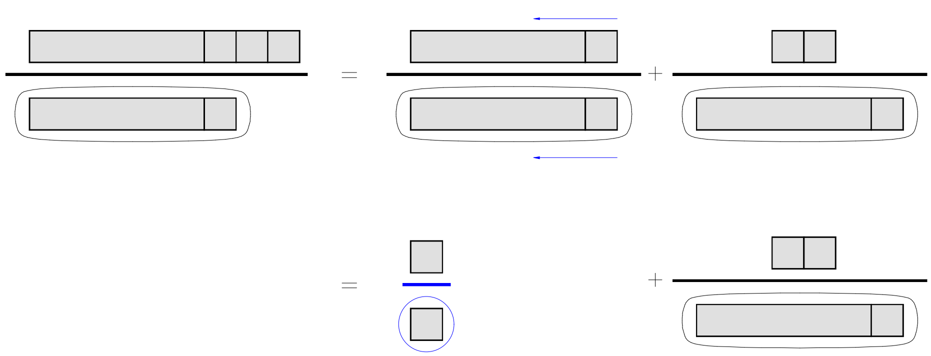
This process of collapsing, or reducing, works because fractions (rational expressions) compare the pieces on top of the fraction to the pieces on the bottom of the fraction. If the top and bottom rectangles have the same length, then the ratio (fraction) of their sizes is the same as the ratio (fraction) of their heights. Collapsing (reducing) is the same as just comparing the heights of the rectangles.

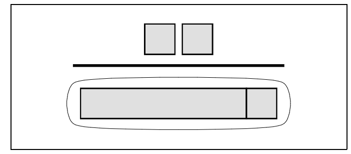
If, when adjusting the top of a fraction (by adding equal numbers of positive and negative chips), there are some pieces left over which do not match the length of the bottom of the fraction, these pieces are the **remainder**. They must be left on top of the original denominator, as shown in the next two examples:

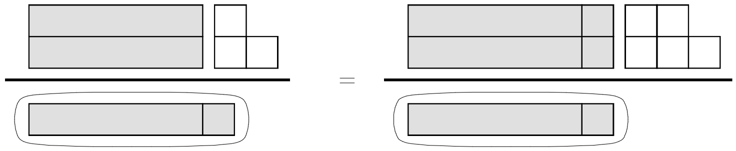
$$\frac{x+3}{x+1} = \frac{x+1+2}{x+1}$$

$$\frac{\cancel{x+1}}{\cancel{x+1}} + \frac{2}{x+1}$$

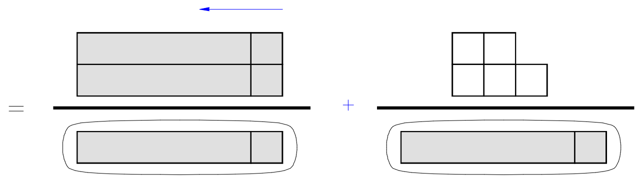
$$1 + \frac{2}{x+1}$$



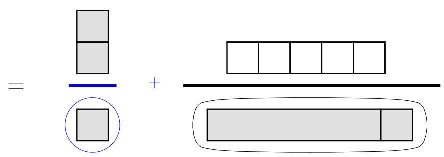




$$\frac{2x-3}{x+1} = \frac{2x+2-5}{x+1}$$



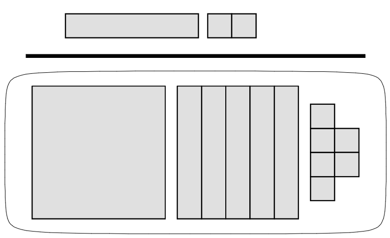
$$\frac{2(x+1)}{x+1} + \frac{^-5}{x+1}$$



$$2 + \frac{^-5}{x+1}$$

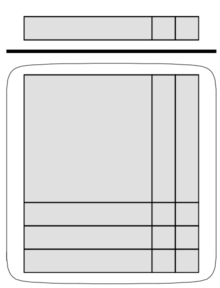
Numerators and Denominators with x^2

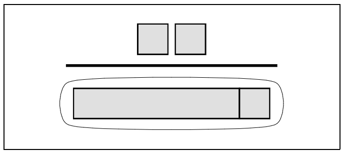
Consider the following rational expression:



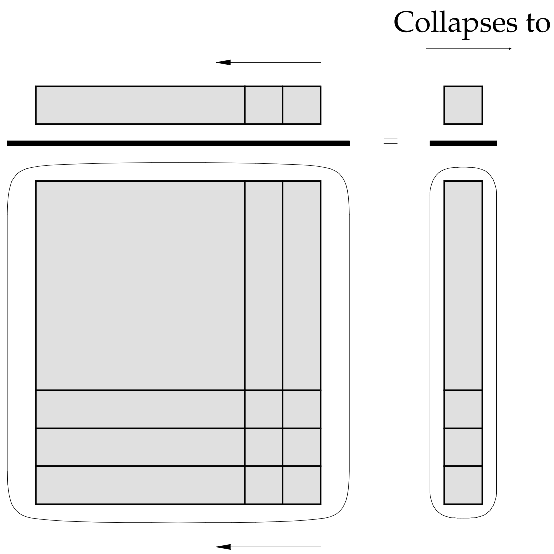
$$\frac{x+2}{x^2+5x+6}$$

Can we simplify this expression? First we must see if the denominator can be made into a rectangle (factored); this can be done in the following way:



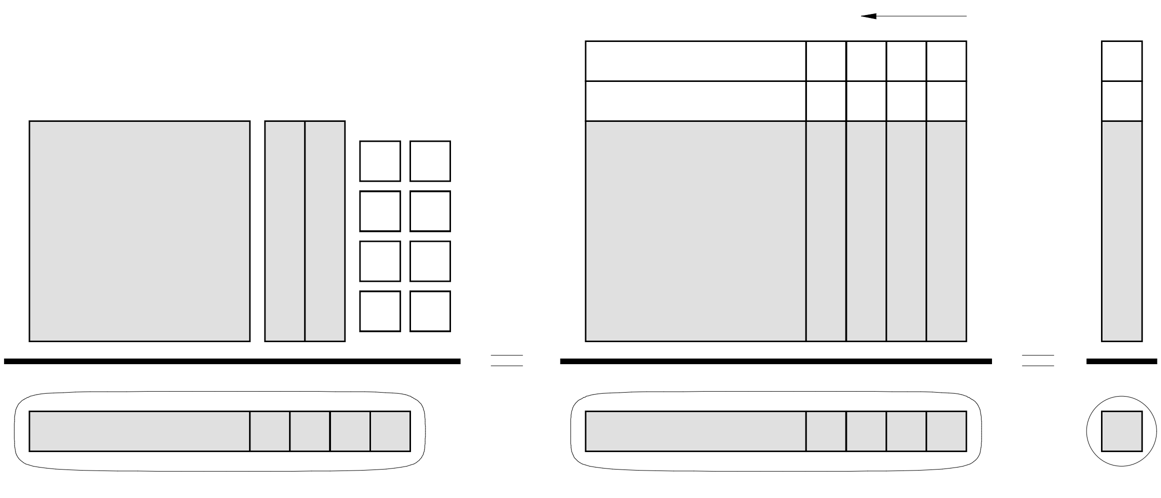


The top and bottom of the expression have the same length, so we can collapse (reduce) both the top and bottom:



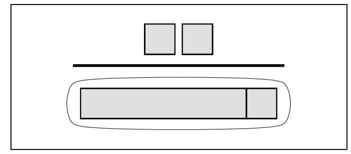
$$\frac{x+2}{x^2+5x+6} = \frac{1\cancel{(x+2)}}{(x+3)\cancel{(x+2)}} = \frac{1}{x+3} \qquad (x+2 \text{ cancels})$$

This is our reduced result. Here is another example:

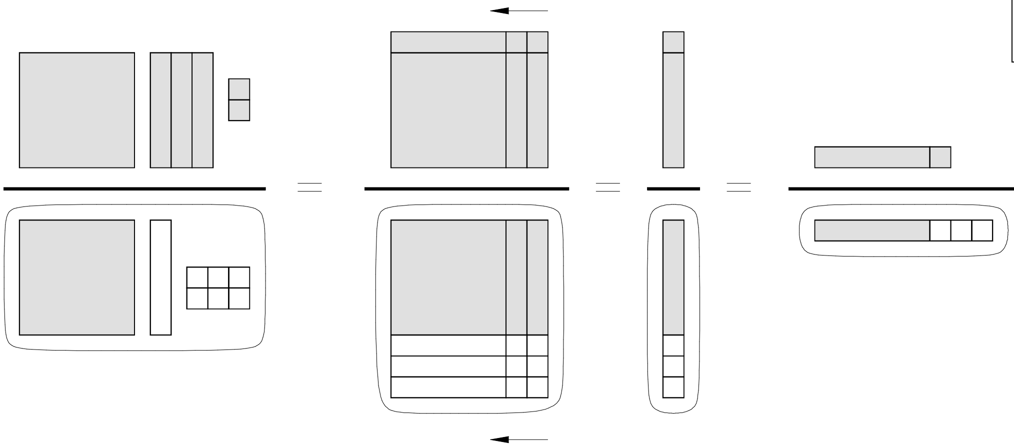


$$\frac{x^2+2x-8}{x+4} = \frac{(x-2)\cancel{(x+4)}}{1\cancel{(x+4)}} = \frac{x-2}{1} = x-2$$

When the denominator reduces to one chip, standing for one unit, we don't have to write the denominator because the ratio of any quantity to one is exactly that quantity. In other words, any quantity divided by one is unchanged.

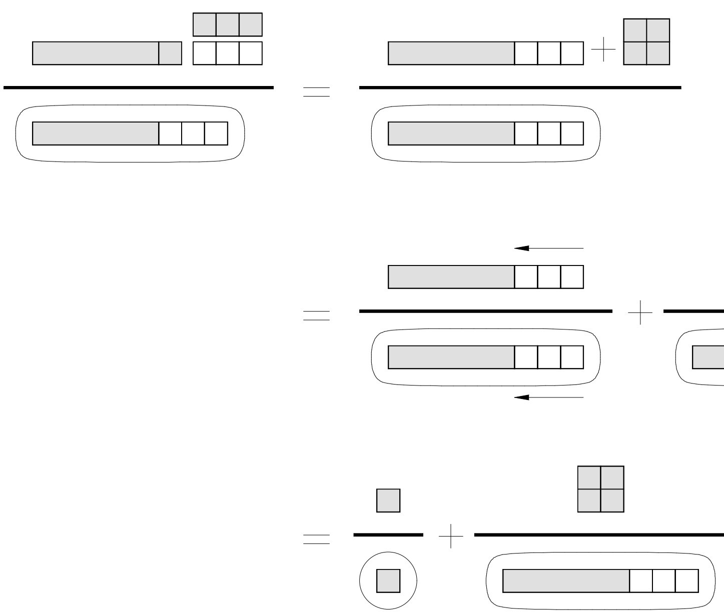


Here is a third problem of this type:



$$\frac{x^2+3x+2}{x^2-x-6} = \frac{\cancel{(x+2)}(x+1)}{\cancel{(x+2)}(x-3)} = \frac{x+1}{x-3}$$

This can be converted further as follows:

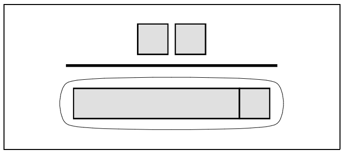


$$1 + \frac{4}{x-3}$$

When to Stop

The two expressions derived for the last problem are equivalent:

$$\frac{x+1}{x-3} = 1 + \frac{4}{x-3}$$



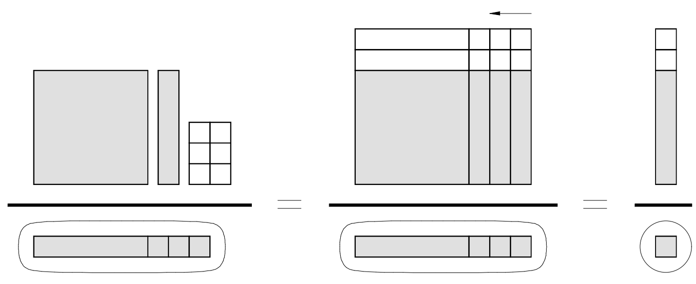
Which is the better way to write the answer?
One method is to stop reducing an expression before we are left with a remainder, at the last step when the expression can be written as a single fraction:

$$\text{Answer \#1: } \frac{x+1}{x-3}$$

But this answer is also correct:

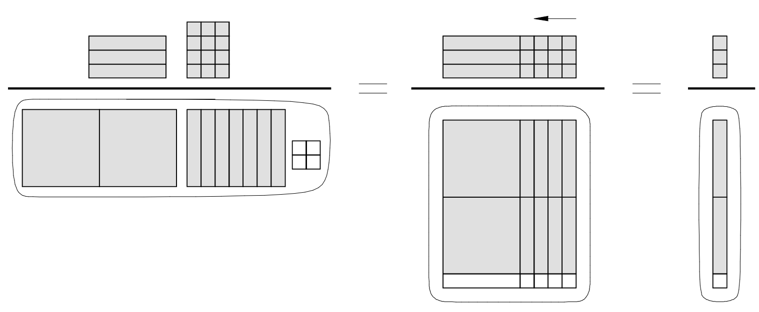
$$\text{Answer \#2: } 1+\frac{4}{x-3}$$

We often stop with answer #1 because this expression, having only one fraction, is easier to work with than answer #2, which has two different terms. To illustrate the these results, we will reduce the following examples:



$$\frac{x^2+x-6}{x+3} = \frac{(x+3)(x-2)}{1(x+3)} = \frac{x-2}{1} = x-2$$

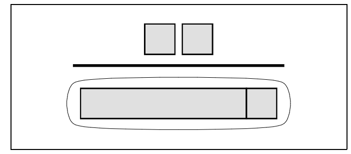
Example 1:

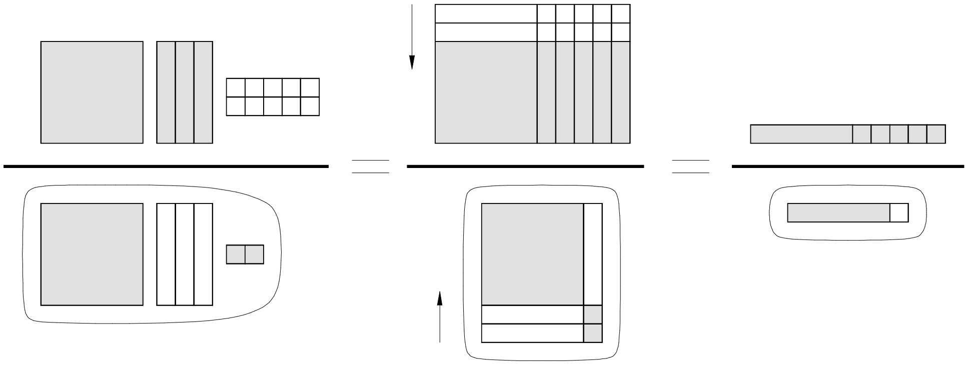


$$\frac{3x+12}{2x^2+7x-4} = \frac{3(x+4)}{(2x-1)(x+4)} = \frac{3}{2x-1}$$

Example 2:

Example 3:





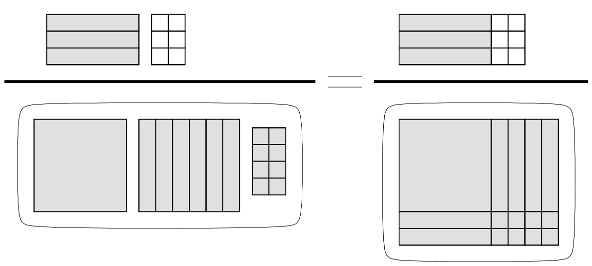
$$\frac{x^2+3x-10}{x^2-3x+2} = \frac{(x-2)(x+5)}{(x-2)(x-1)} = \frac{x+5}{x-1}$$

We will stop here, although $1+\frac{6}{x-1}$ is also correct.

When There are No Common Factors

If the numerator and denominator of a rational expression have no common factors, then we can factor both the numerator and the denominator as much as possible and leave them in factored form without reducing.

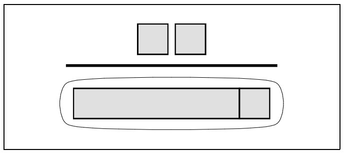
For example:



$$\frac{3x-6}{x^2+6x+8} = \frac{3(x-2)}{(x+2)(x+4)}$$

Exercises

Reduce:



1. $\dfrac{2x-6}{x-3}$

2. $\dfrac{x+5}{3x+15}$

3. $\dfrac{3x+6}{2x+4}$

4. $\dfrac{x^2-4x}{3x}$

5. $\dfrac{x+3}{3x+9}$

6. $\dfrac{2x+18}{x^2+9x}$

Reduce to an expression with a remainder (add and subtract equal amounts if necessary):

7. $\dfrac{x+5}{x+2}$

8. $\dfrac{x-3}{x-1}$

9. $\dfrac{x+2}{x+5}$

10. $\dfrac{2x+3}{2x-5}$

11. $\dfrac{3x-5}{x-3}$

12. $\dfrac{2x+6}{x+2}$

Factor. Reduce if possible:

13. $\dfrac{2x+6}{x^2+x-6}$

14. $\dfrac{x^2+3x+2}{x^2-3x-10}$

15. $\dfrac{3x-12}{x^2+5x+6}$

16. $\dfrac{x^2+6x+5}{3x+3}$

17. $\dfrac{2x^2-x-6}{x^2-7x+10}$

18. $\dfrac{3x}{x^2-7x}$

19. $\dfrac{x^2+x-12}{3x^2+5x-2}$

20. $\dfrac{2x^2-3x}{2x^2+7x-15}$

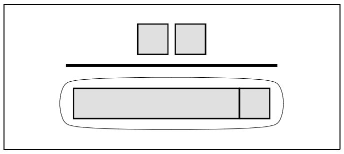
$$
\begin{array}{r}
14\,{}^{15}\!/_{17} \\
17\,\overline{)\,253} \\
-\underline{17} \\
83 \\
-\underline{68} \\
15
\end{array}
$$

$$\frac{253}{17} = 14\,\frac{15}{17}$$

$$(17 \cdot 14\,{}^{15}\!/_{17}) = (17 \cdot 14) + (17 \cdot {}^{15}\!/_{17})$$

$$
\begin{array}{r}
14 \\
\times\ 17 \\
\hline
98 \\
+\ 140 \\
\hline
238 \\
+\ 15 \\
\hline
253
\end{array}
$$

$$(17 \cdot {}^{15}\!/_{17}) = 15$$

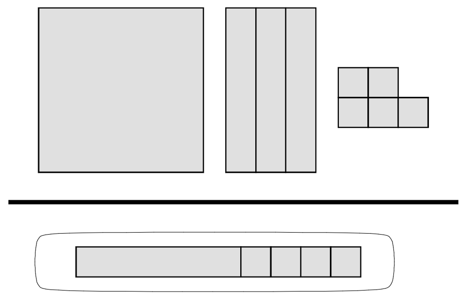


Section 3
Division Using Chips

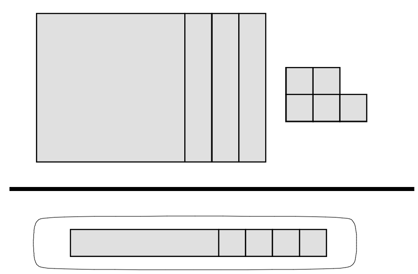
Division with Numbers

If we have a fraction containing whole numbers, with the numerator larger

$$\frac{x^2 + 3x + 5}{x + 4}$$

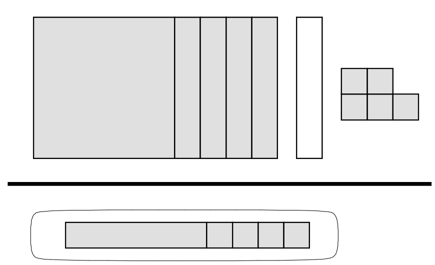


than the denominator we know that we can treat the fraction as a division problem; we divide the bottom number into the top number to get a simplified answer. For example, consider the fraction $^{253}/_{17}$. We can simplify this fraction by doing long division:

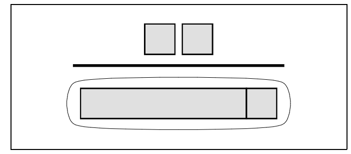


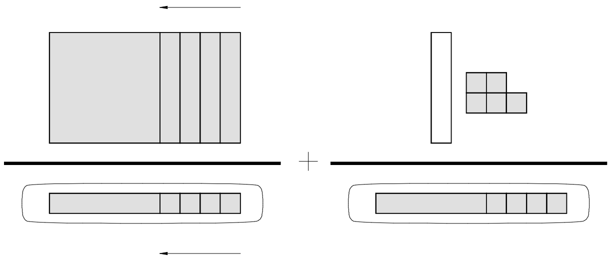
Once we have done this long division, we can check our result by multiplying the divisor (17) times the answer ($14\ ^{15}/_{17}$), to see if we get the original numerator (253).

$$\frac{x^2 + 4x + (-x) + 5}{x + 4}$$

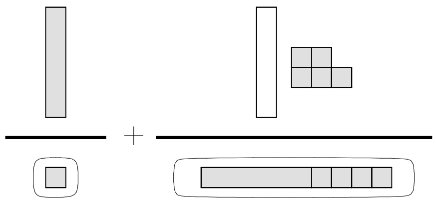


Here is the check:



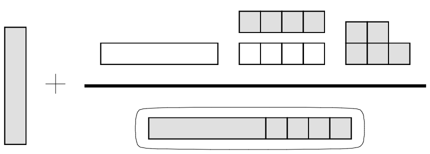


$$\frac{x(x+4)}{1(x+4)} + \frac{-x+5}{x+4}$$



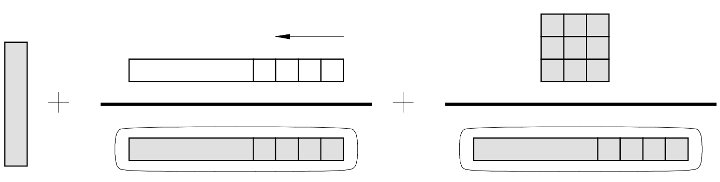
$$\frac{x}{1} + \frac{-x+5}{x+4}$$

Division with Unknowns



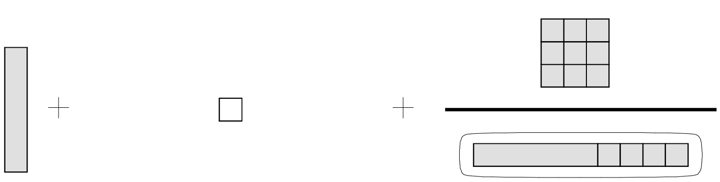
$$x + \frac{-x-4+9}{x+4}$$

There is a similar process for reducing rational expressions. If the numerator



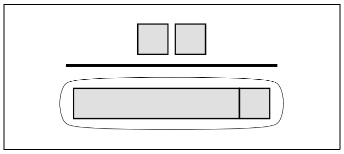
$$x + \frac{(-1)(x+4)}{1(x+4)} + \frac{9}{x+4}$$

$$x + \frac{-1}{1} + \frac{9}{x+4}$$



$$x + {}^{-}1 + \frac{9}{x+4}$$

of a rational expression is of the same or higher order than the denominator,



we can reduce the rational expression through a process of long division. The method is similar to what we have done already in section 1 of this chapter.

For example:

$$\frac{x^2 + 3x + 5}{x + 4}$$

We need to arrange the terms of the numerator so that they are in groups which share the dimension of length with the denominator. We can do this, you will recall, by adding equal numbers of positive and negative pieces until we get terms which will work. Begin with the x^2 and the x terms of the numerator.

These first two terms can make a rectangle having the length of the denominator $(x + 4)$ if we add a $+x$ and a $-x$, and group the $+x$'s with the x^2 pieces.

This gives:

Now that the first two terms of the numerator share the same length as the denominator, we separate the expression into two fractions. The first fraction reduces (collapses) as shown:

To repeat the process, we add plus and minus units to the second fraction's numerator, making another fraction which reduces:

$$\frac{x^2 + 4x}{x + 4} + \frac{{}^-1x + 5}{x + 4}$$

$$\frac{x\cancel{(x + 4)}}{1\cancel{(x + 4)}} + \frac{{}^-1x + 5}{x + 4}$$

The numerator of the final fraction has no x-bars, so it cannot match the length of the denominator. This is the remainder; it remains a fraction and cannot be reduced. We now have two terms and a small fraction remainder.

Let's follow through the steps of the long division process.

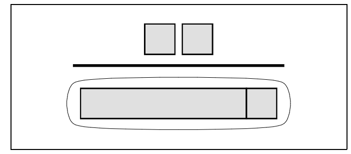
$$\frac{x^2 + 3x + 5}{x + 4}$$

We look at the first terms of the numerator and denominator. The numerator has an x^2, which is x times the first term (x) of the denominator. We build a rectangle with one dimension as the length of the denominator $(x + 4)$, and the other dimension of x (which gives us the x^2), making the rectangle

$$(x)(x + 4) = x^2 + 4x.$$

We add a $+x$ and a $-x$ to the numerator, giving us $x^2 + 4x$:

$$\frac{x^2 + 4x - 1x + 5}{x + 4}$$



The first two terms have one factor (one dimension) which matches the denominator, so we split the expression into two fractions. We then factor and reduce the first fraction:
The first terms of both the numerator and denominator of the second fraction contain an x. On the top, ($-x$) is ($^-1$) times x; in order to have a factor of (x + 4) in the numerator we will need a numerator having the terms $^-1(x + 4)$ which is $-x - 4$. To get this we add a $^-4$ and a $^+4$ to the numerator, and split off a second fraction which will reduce, leaving a remainder.

$$x + \frac{^-x + 5}{x + 4} = x + \frac{^-x - 4 + 4 + 5}{x + 4}$$

$$= x + \frac{(^-1)(x + 4)}{(1)(x + 4)} + \frac{9}{x + 4}$$

$$= x + (^-1) + \frac{9}{x + 4}$$

This is our answer.
To summarize:

(x + 4) goes into ($x^2 + 3x + 5$), $x + {^-1} + \frac{9}{x + 4}$ times.

(We will show how this process is the same as regular numerical long division later in this section.)

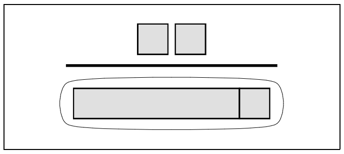
To get a feel for what we have done, let's check our results for a specific value of x, to show that our solution gives the correct answer. If we choose $x = 3$, then we can substitute this value for x into the original expression and into our answer to see if the results are the same. Our result says

$$\frac{x^2 + 3x + 5}{x + 4} = x + {^-1} + \frac{9}{x + 4}$$

Substituting $x = 3$, we get the following:

$$\frac{x^2 + 3x + 5}{x + 4} = \frac{(3)^2 + 3(3) + 5}{3 + 4}$$

$$= \frac{9 + 9 + 5}{7}$$



$$= \frac{23}{7}$$

$$= \mathbf{3\frac{2}{7}} \quad \leftarrow$$

$$x + {}^{-}1 + \frac{9}{x+4} = 3 - 1 + \frac{9}{3+4}$$

$$= 2 + \frac{9}{7}$$

$$= 2 + 1\frac{2}{7}$$

$$= \mathbf{3\frac{2}{7}} \quad \leftarrow$$

We have confirmed that our long division process was correct. When x is 3,

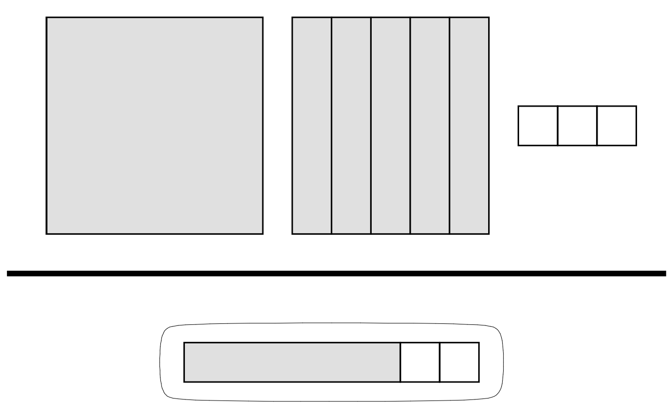
$$\text{Is } (x+4) \text{ times } x + {}^{-}1 + \frac{9}{x+4} \text{ equal to } (x^2 + 3x + 5)?$$

$$(x+4)\left(x + {}^{-}1 + \frac{9}{x+4}\right) = (x+4)(x + {}^{-}1) + \frac{\cancel{(x+4)}}{1} \cdot \frac{9}{\cancel{(x+4)}}$$
$$= x^2 + {}^{-}x + 4x - 4 + 9$$
$$= x^2 + 3x - 4 + 9$$
$$= x^2 + 3x + 5$$

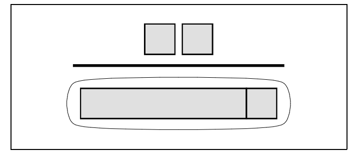
we get the same value for the original expression and for the expression after long division.

Now you try it for a different value of x. Does it work every time? How

$$\frac{x^2 + 5x - 3}{x - 2}$$

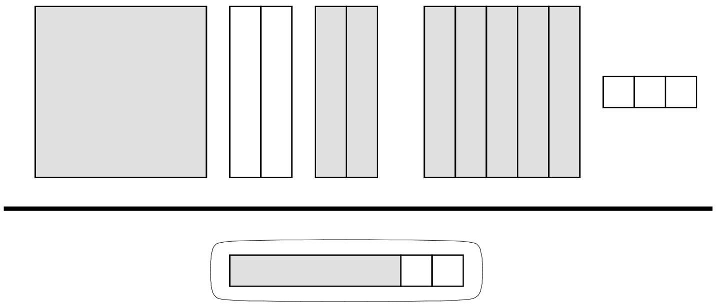


about when x is negative? Let $x = {}^{-}2$:

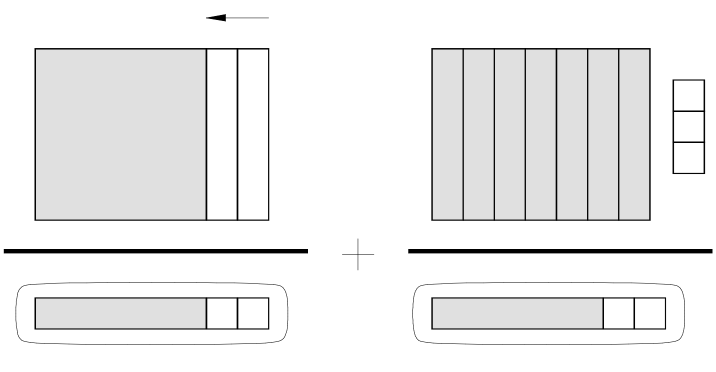


$$\frac{x^2+3x+5}{x+4} = \frac{(^-2)^2+3(^-2)+5}{(^-2)+4}$$

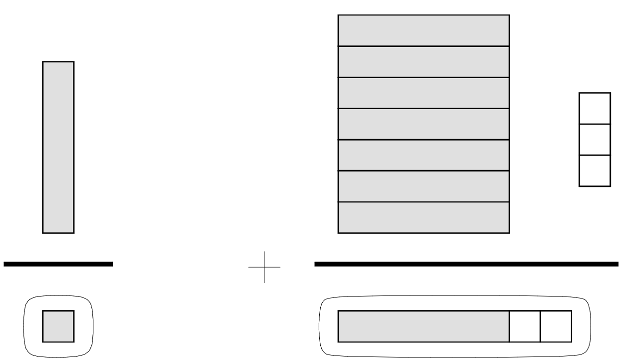
$$= \frac{4-6+5}{^+2}$$



$$\frac{x^2-2x+7x-3}{x-2}$$



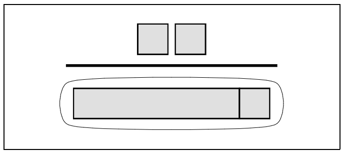
$$\frac{x^2-2x}{x-2}+\frac{7x-3}{x-2}$$



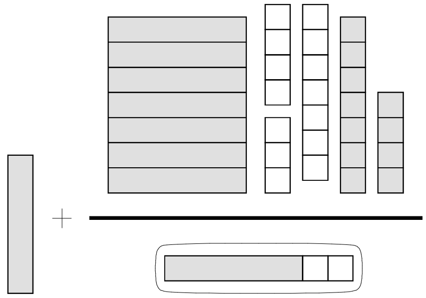
$$\frac{x(\cancel{x-2})}{1(\cancel{x-2})}+\frac{7x-3}{x-2}$$

$$= x+\frac{7x-3}{x-2}$$

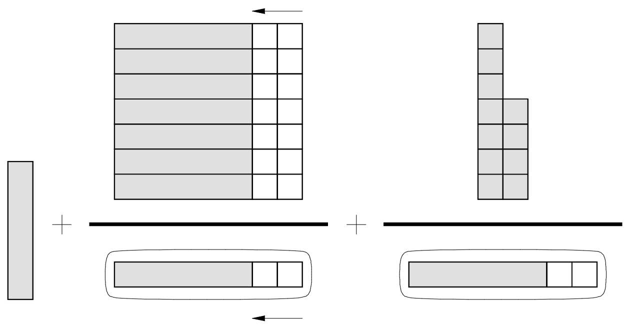
$$= \frac{3}{2} = \mathbf{1\,\frac{1}{2}} \quad \leftarrow$$



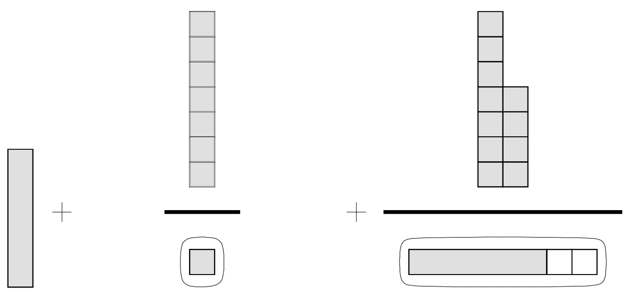
$$x + \frac{7x - 3 - 11 + 11}{x - 2}$$



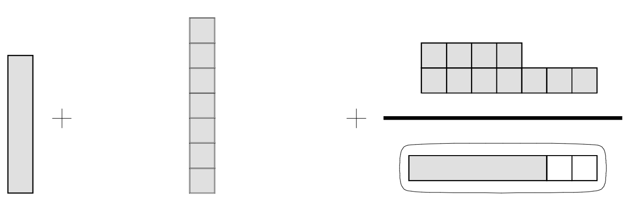
$$x + \frac{7x - 14}{x - 2} + \frac{11}{x - 2}$$

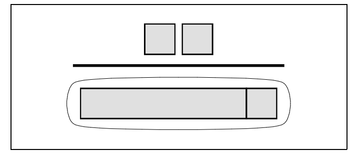


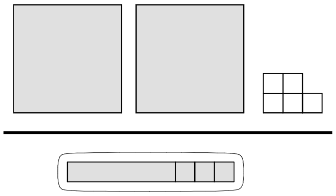
$$x + \frac{7\cancel{(x-2)}}{\cancel{(x-2)}} + \frac{11}{x - 2}$$



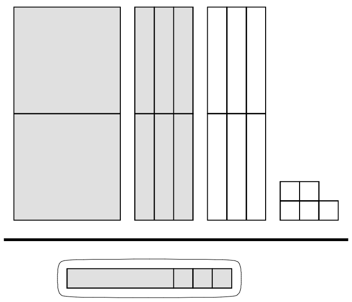
$$x + 7 + \frac{11}{x - 2}$$



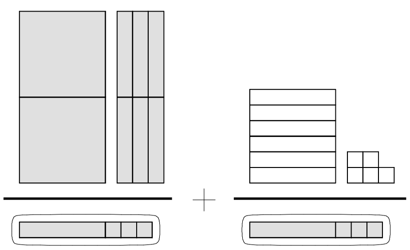




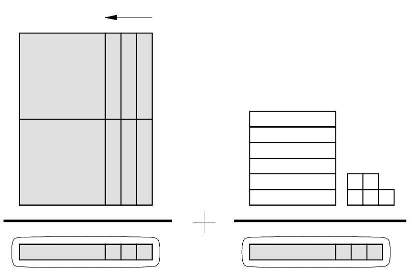
$$\frac{2x^2 - 5}{x + 3}$$



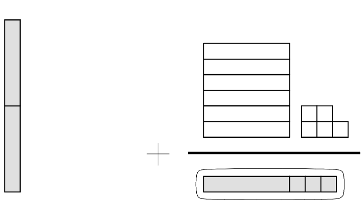
$$\frac{2x^2 + 6x - 6x - 5}{x + 3}$$



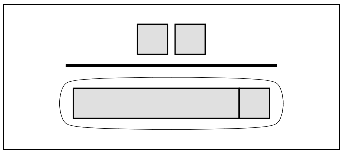
$$\frac{2x^2 + 6x}{x + 3} + \frac{^{-}6x - 5}{x + 3}$$



$$\frac{2x\cancel{(x + 3)}}{\cancel{(x + 3)}} + \frac{^{-}6x - 5}{x + 3}$$



$$2x + \frac{^{-}6x - 5}{x + 3}$$

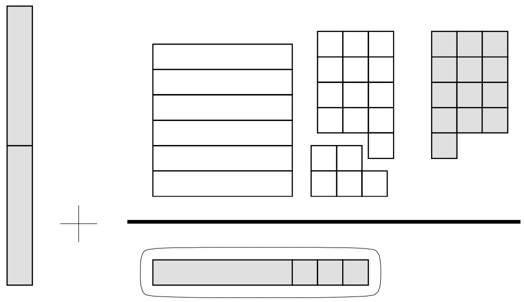


$$x - 1 + \frac{9}{x+4} = {}^{-}2 - 1 + \frac{9}{{}^{-}2 + 4}$$

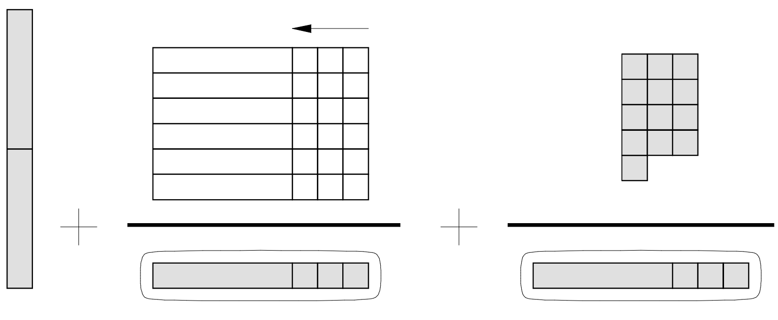
$$= {}^{-}3 + \frac{9}{2}$$

$$= {}^{-}3 + 4\,\frac{1}{2}$$

$$2x + \frac{{}^{-}6x - 5 - 13 + 13}{x+3}$$



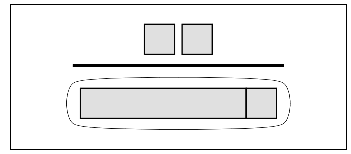
$$2x + \frac{{}^{-}6x - 18}{x+3} + \frac{13}{x+3}$$



$$2x + \frac{{}^{-}6(x+3)}{1(x+3)} + \frac{13}{x+3}$$

$$= 2x + {}^{-}6 + \frac{13}{x+3}$$

$$= \mathbf{1\,\frac{1}{2}} \quad \leftarrow$$



One more way to check the results of this long division is to multiply the divisor ($x + 4$) times the answer to see if we get the original dividend:
So the solution works for any value of x.

Let's do another example. Use long division to reduce the following rational expression:

First, we need to make a rectangle in the numerator which uses the x^2, and which has length ($x - 2$), the length of the denominator. This will require

$$x(x-2) = x^2 - 2x$$

Since the numerator doesn't have any negative x-bars, we must add $^{-}2x$ and $^{+}2x$ to the numerator. Then we break off the first two terms into a separate fraction which can reduce:
In the remaining fraction, we wish to make a rectangle (x - 2) in length, while using the 7 x-bars in the numerator. For the numerator, this will require

$$7(x-2) = 7x - 14$$

Since we have $7x - 3$, we must add a $^{-}11$ and a $^{+}11$ to the numerator:
The result is

$$\frac{x^2 + 5x - 3}{x - 2} = x + 7 + \frac{11}{x - 2}$$

A third and final example will show how this method even fills in for missing terms in the numerator. Consider this expression:
We need a rectangle $2x$ by ($x + 3$), or $2x^2 + 6x$. So we add $+6x$ and $^{-}6x$ to the numerator:
This breaks into two fractions, the first of which reduces:

The numerator of the second fraction will now need a rectangle which is ($x + 3$) long, and $^{-}6$ high, or

$$^{-}6(x+3) = {}^{-}6x - 18$$

This requires adding $^{-}13$ and $^{+}13$ to the numerator, and then breaking it into two fractions. The first fraction will reduce, and the other fraction is the remainder.
The result is:

$$\frac{2x^2 - 5}{x + 3} = 2x + {}^{-}6 + \frac{13}{x + 3}$$

Exercises

Use chips or drawings to complete the following reductions by long division:

1. $$\frac{x^2 + 6x - 4}{x + 2}$$

2. $$\frac{x^2 - 5x + 3}{x + 1}$$

3. $$\frac{2x^2 + 3x - 6}{x - 3}$$

4. $$\frac{3x^2 - 5}{x + 1}$$

5. $$\frac{x^2 - 4x}{x - 2}$$

Beginning with the given fraction,

$$\frac{253}{17}$$

Place the numerator under the division sign, and the denominator in front as the divisor.

$$17\overline{)\;2\;5\;3}$$

Now look at how many times the 17 goes into the 25 and write the first partial answer (1) above the division bar:

$$\begin{array}{r} \mathbf{1} \\ \mathbf{17}\overline{)\;\mathbf{2}\;\mathbf{5}\;3} \end{array}$$

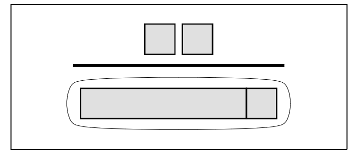
6. $$\frac{2x^2 + 6x - 5}{x - 2}$$

Next multiply the first partial answer (the 1) times the 17 and write the answer (also 17) below the 25.

$$\begin{array}{r} 1 \\ 17\overline{)\;2\;5\;3} \\ \underline{1\;7} \end{array}$$

To find out what remains, subtract the bottom 17 from the 25, giving 8, and bring down the 3:

$$\begin{array}{r} 1 \\ 17\overline{)\;2\;5\;3} \\ \underline{-\,1\;7} \\ 8\;3 \end{array}$$



$$\begin{array}{r} 14 \\ 17 \overline{)253} \\ -17 \\ \hline 83 \end{array}$$

Now look at the divisor (17) and at the 83, and decide how many times 17 can go into 83. (The answer is 4.) Place this second partial answer above the division bar as shown.

$$\begin{array}{r} 14 \\ 17 \overline{)253} \\ -17 \\ \hline 83 \\ -\ 68 \\ \hline 15 \end{array}$$

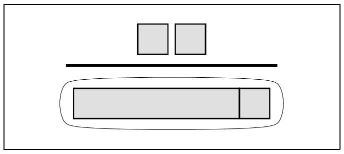
Multiply the second partial answer (the 4) times the 17, and put the result (68) under the 83. Subtract the 68 from the 83, leaving 15.

7. $\dfrac{x^2 - 5x + 1}{x + 3}$

$$\begin{array}{r} 14\tfrac{15}{17} \\ 17 \overline{)253} \\ -17 \\ \hline 83 \\ -\ 68 \\ \hline 15 \end{array}$$

Finally, since 17 will not divide into 15, this is our remainder, which is written as the fraction $15/17$.

8. $\dfrac{2x^2 - x}{x + 5}$



$$\frac{x^2 + 3x + 5}{x + 4}$$

Begin with the given rational expression, and place the numerator under the division bar, with the denominator in front as the divisor.

$$x + 4 \overline{)\; x^2 + 3x + 5}$$

Now look at the first term of both expressions, (the x and the x^2) and decide how many times x will go into x^2 ? Write the answer (x) on top of the division bar.

$$\begin{array}{r} x \\ x + 4 \overline{)\; x^2 + 3x + 5} \end{array}$$

Next multiply this first partial answer (the top x) times ($x + 4$), and put the result ($x^2 + 4x$) below the $x^2 + 3x$.

$$\begin{array}{r} x \\ x + 4 \overline{)\; x^2 + 3x + 5} \\ x^2 + 4x \end{array}$$

To find out what will remain, we must subtract the $x^2 + 4x$ from the $x^2 + 3x$. (The easiest way to do this is to change the signs on both bottom terms, the x^2 and the $4x$, and then add the results to the columns above them as shown.) Bring down the +5.

$$\begin{array}{r} x \\ x + 4 \overline{)\; x^2 + 3x + 5} \\ \underline{-x^2 + {}^{-}4x} \\ 0 + {}^{-}1x + 5 \end{array}$$

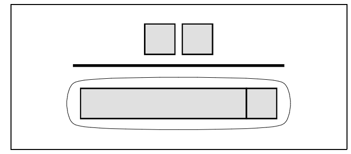
change signs
and add

Now look at the first term of the divisor ($x + 4$) and the $^{-}1x + 5$ and decide what must multiply the x to give $^{-}1x$. The required multiplier is $^{-}1$, which is written above the division bar.

$$\begin{array}{r} x - 1 \\ x + 4 \overline{)\; x^2 + 3x + 5} \\ \underline{-x^2 + {}^{-}4x} \\ {}^{-}1x + 5 \end{array}$$

Multiply this second partial answer (the $^{-}1$) times the divisor ($x + 4$) and write the result under the $^{-}1x + 5$:

$$\begin{array}{r} x - 1 \\ x + 4 \overline{)\; x^2 + 3x + 5} \\ \underline{-x^2 + {}^{-}4x} \\ {}^{-}1x + 5 \\ \underline{{}^{-}1x - 4} \end{array}$$



$$\begin{array}{r r r r r r r r}
 & & & x & - & 1 & & \\
\hline
x + 4 &) & & x^2 & + & 3x & + & 5 \\
 & & & -x^2 & + & -4x & & \\
\hline
 & & & & & -1x & + & 5 \\
\text{change signs and add} & & & & & x & + & 4 \\
\hline
 & & & & & 0 & + & 9
\end{array}$$

To find out how much remains, subtract the $-x - 4$ from the $^-1x + 5$. (Again this is done by changing the signs on both terms of the $-x - 4$, making them $+x + 4$, and adding the result to the columns above.)

$$\begin{array}{r r r r r r r r}
 & & & x & - & 1 & + & 9/(x+4) \\
\hline
x + 4 &) & & x^2 & + & 3x & + & 5 \\
 & & & -x^2 & + & -4x & & \\
\hline
 & & & & & -1x & + & 5 \\
 & & & & & x & + & 4 \\
\hline
 & & & & & & & 9/(x+4)
\end{array}$$

The 9 which is left is the remainder, and it is written as the numerator of a fraction having the divisor $(x + 4)$ as the denominator. This fraction is added on to the answer.

n **4**

Division

$$\frac{x^2 + 5x + 3}{x - 2}$$

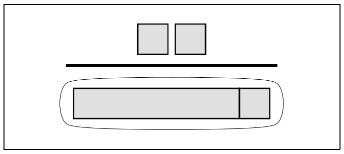
Rewrite as a division

$$\begin{array}{r r r r r r}
 & & x & & & \\
\hline
x - 2 &) & x^2 & + 5x & + & 3 \\
 & & x^2 & - 2x & &
\end{array}$$

Look at first terms and put the partial answer up. Multiply through.

Short-Cut Method of Long Division

We will now look at the regular long division of numbers. Our purpose is



Change signs and add. Bring down the third term.

$$
\begin{array}{r|l}
 & x \\
\cline{2-2}
x - 2 & x^2 + 5x + 3 \\
 & -\,x^2 + 2x \\
\cline{2-2}
 & 0 + 7x - 3
\end{array}
$$

Look at the first terms and write the second partial answer. Multiply through.

$$
\begin{array}{r|l}
 & x + 7 \\
\cline{2-2}
x - 2 & x^2 + 5x + 3 \\
 & -\,x^2 + 2x \\
\cline{2-2}
 & \quad + 7x - 3 \\
 & \quad + 7x - 14
\end{array}
$$

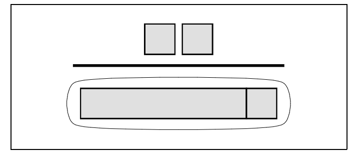
Change signs and add.

$$
\begin{array}{r|l}
 & x + 7 \\
\cline{2-2}
x - 2 & x^2 + 5x + 3 \\
 & -\,x^2 + 2x \\
\cline{2-2}
 & \quad 7x - 3 \\
 & \quad -7x + 14 \\
\cline{2-2}
 & \quad 0 + 11
\end{array}
$$

Form a fraction from the remainder.

$$
\begin{array}{r|l}
 & x + 7 + 11/_{x-2} \\
\cline{2-2}
x - 2 & x^2 + 5x + 3 \\
 & -\,x^2 + 2x \\
\cline{2-2}
 & \quad 7x - 3 \\
 & \quad -7x + 14 \\
\cline{2-2}
 & \quad 0 + 11/(x-2)
\end{array}
$$

to discover how to do the same type of long division with *rational expressions*:



$$\frac{2x^2 - 5}{x + 3}$$

$$x + 3 \overline{\smash{\big)}\, 2x^2 + 0x - 5}$$

Rewrite as a division problem; put a column in for x's by writing the dividend $2x^2 + 0x - 5$

$$\begin{array}{r l}
 & 2x \\
x + 3 \big) & \overline{2x^2 + 0x - 5} \\
 & \underline{2x^2 + 6x}
\end{array}$$

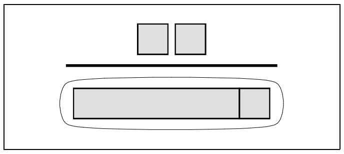
Look at the first terms and write a first partial answer. Multiply through.

$$\begin{array}{r l}
 & 2x \\
x + 3 \big) & \overline{2x^2 \; + \; 0x \; - 5} \\
 & \underline{^-2x^2 + {^-6x}} \\
 & 0 \;\; + {^-6x} - 5
\end{array}$$

Change the signs and add. Bring down the third term.

$$\begin{array}{r l}
 & 2x \; - \; 6 \\
x + 3 \big) & \overline{2x^2 \; + \; 0x \; - 5} \\
 & \underline{^-2x^2 + {^-6x}} \\
 & \qquad\quad {^-6x} - 5 \\
 & \qquad\quad \underline{^-6x - 18}
\end{array}$$

Look at the first terms and write a second partial answer. Multiply through.



Change signs and add. Form the remainder into a fraction.

$$
\begin{array}{r|l}
 & 2x \quad - \quad 6 \quad + {}^{13}\!/\!_{(x+3)} \\
\hline
x + 3 & 2x^2 \quad + \quad 0x \quad - \quad 5 \\
 & {}^{-}2x^2 \quad + \quad {}^{-}6x \\
 & \qquad\qquad {}^{-}6x \quad - \quad 5 \\
 & \qquad\qquad {}^{+}6x \quad + \quad 18 \\
 & \qquad\qquad 0 \quad + \, {}^{13}\!/\!_{x+3}
\end{array}
$$

Dividing Rational Expressions

The long division method for rational expressions is nearly identical with that for numbers as shown above. We will illustrate using our first example from the last section. As you work through the problems, refer back to the pictures of the method we developed earlier for dividing rational expressions so you can see where the steps come from.

Example 1:

$$\frac{x^2 + 3x + 5}{x + 4}$$

The final result is:

$$\frac{x^2 + 3x + 5}{x + 4} = x - 1 + \frac{9}{x + 4}$$

Now we will work through this method for the other two examples.

Example 2:

The final answer:

$$\frac{x^2 + 5x - 3}{x - 2} = x + 7 + \frac{11}{x - 2}$$

Example 3:

The final answer:

$$\frac{2x^2 - 5}{x + 3} = 2x - 6 + \frac{13}{x + 3}$$

Exercises

Reduce the following by the long division method:

1. $\dfrac{x^2+3x-5}{x-2}$

2. $\dfrac{2x^2-3x-1}{x+3}$

3. $\dfrac{x^2+7x+2}{x+1}$

4. $\dfrac{3x^2+5}{x-4}$

5. $\dfrac{x^2-2x}{x+2}$

6. $\dfrac{5x^2+7x-11}{x-5}$

7. $\dfrac{x^2-2x+5}{x-3}$

8. $\dfrac{2x^2-3x}{x+5}$

9. $\dfrac{x^2-3x-2}{x+1}$

10. $\dfrac{3x^2+2x+1}{x+4}$

Section **5**
Multiplication

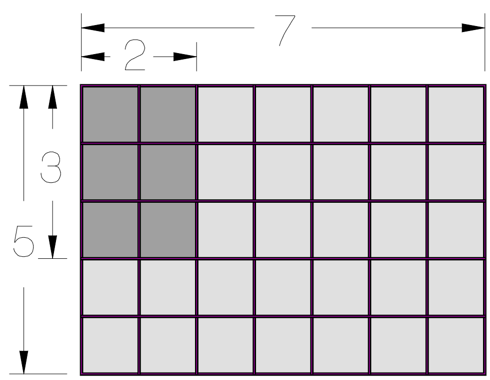
Multiplying Fractions

To understand how multiplication works with rational expressions, we need to briefly review multiplication of common fractions. You will remember (from Chapter 4) that when we multiply two fractions

$$\frac{3}{5} \cdot \frac{2}{7}$$

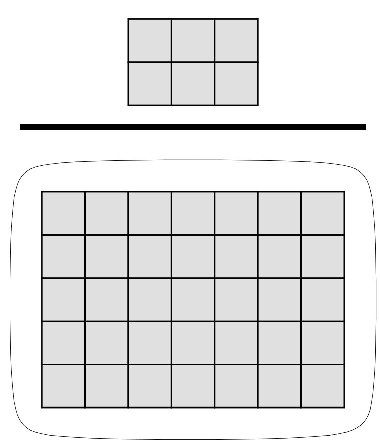
the denominators multiply $(5 \cdot 7)$ to give the total number of pieces needed to make one whole unit; and the numerators multiply $2 \cdot 3$ to tell us how many of these pieces we have.

$$\frac{3}{5} \cdot \frac{2}{7} = \frac{6}{35}$$

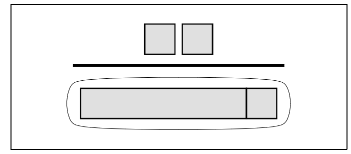


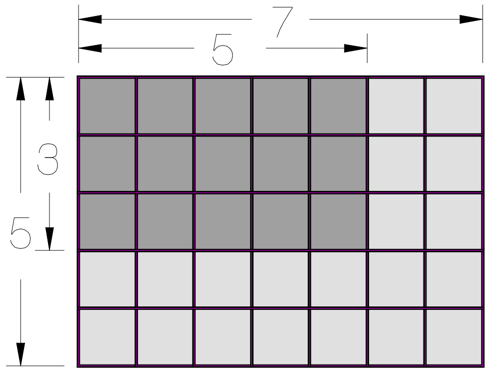
In this example the result is $^6\!/_{35}$, or six thirty-fifth's of one whole unit.

Using our new form of representation this result can also be shown as:



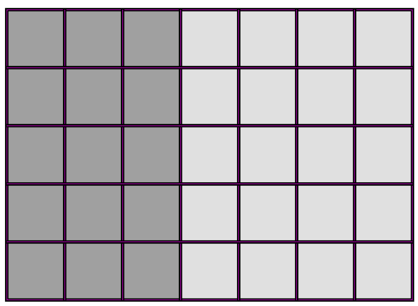
If we had chosen to multiply the fractions $3/5 \cdot 5/7$ our result would have looked like this:





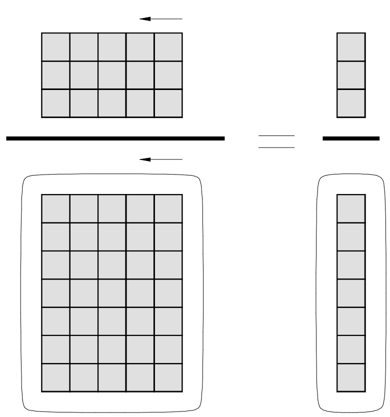
$$\frac{3}{5} \cdot \frac{5}{7} = \frac{15}{35}$$

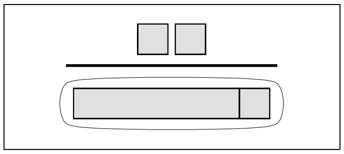
We can see the result is 15 chips out of 35, but the arrangement of the chips can be changed to show how this result can be reduced. 15 out of 35 can also be arranged like this:



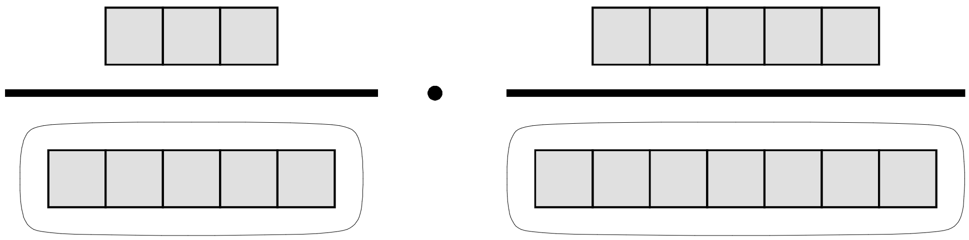
We see 3 shaded columns out of a total of 7 columns, for a reduced result of $3/7$.

In the new representation, this reduction is easily seen, since the rectangles of the numerator and the denominator are the same length in one dimension, which means that they can reduce (collapse), also giving $3/7$.



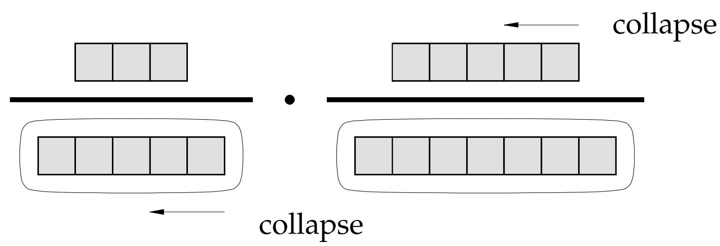


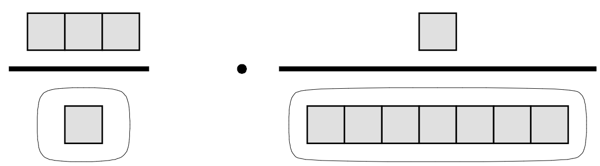
We could have reduced even before we multiplied:

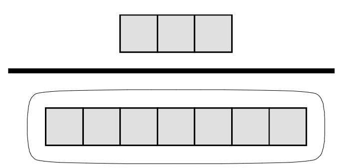


$$\frac{3}{5} \qquad\qquad \frac{5}{7}$$

Since the numerator of each fraction in the multiplication will be one dimension of the final numerator rectangle, and the denominator of each fraction will be one dimension of the final denominator rectangle, the 5 in the second fraction's numerator will reduce (cancel) with the 5 in the first fraction's denominator.





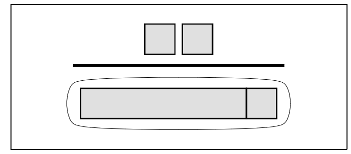


Numerically, we would write this result as

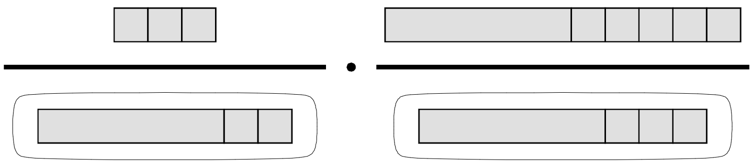
$$\frac{3}{5}\cdot\frac{5}{7} = \frac{3}{\not 5}\cdot\frac{\not 5}{7} = \frac{3}{1}\cdot\frac{1}{7} = \frac{3}{5}$$

(You might wish to review the multiplication and reduction of fractions from Chapter 4 before going on.)

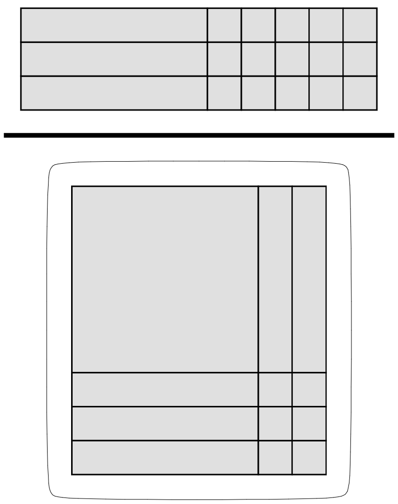
Multiplying Rational Expressions



We are now ready to consider the multiplication of two rational expressions having unknowns. Let's begin with an uncomplicated example:



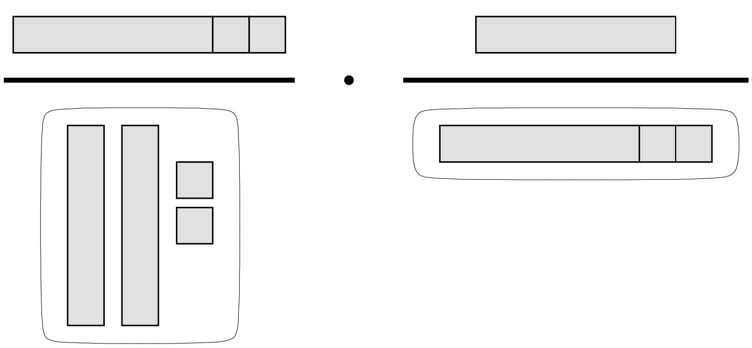
$$\frac{3}{x+2} \cdot \frac{x+5}{x+3}$$



$$\frac{(3)\cdot(x+5)}{(x+2)\cdot(x+3)} = \frac{3x+15}{x^2+5x+6}$$

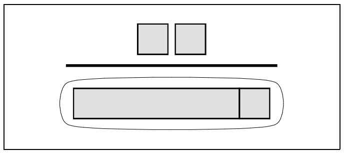
Since none of the dimensions of the numerator and denominator rectangles are the same, this result cannot reduce. *The result written in symbols can be correctly shown in either factored or unfactored form*.

Now let's multiply two other expressions:



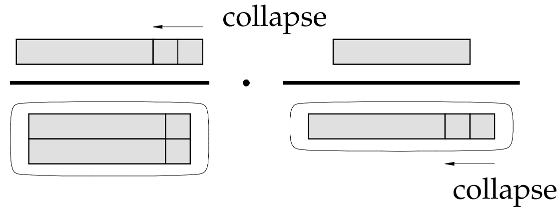
$$\frac{x+2}{2x+2} \cdot \frac{x}{x+2}$$

Remember that we can't cancel pieces; we must make rectangles from the pieces on the top and bottom of each fraction, and if the rectangles have a *dimension* (*factor*) in common on the top and bottom, then that *dimension* can collapse (canceling that *factor*).

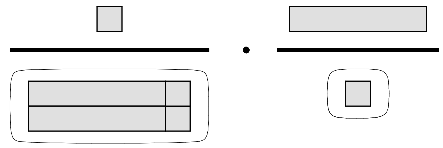


We continue:

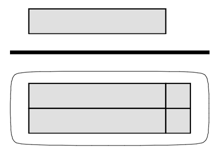
$$\frac{\cancel{(x+2)}}{2(x+1)} \cdot \frac{x}{\cancel{(x+2)}}$$



$$\frac{1}{2(x+1)} \cdot \frac{x}{1}$$



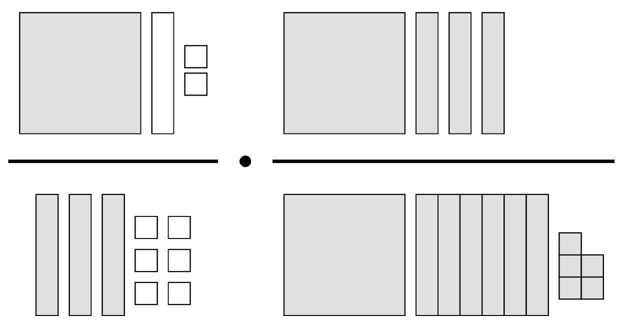
$$\frac{x}{2(x+1)}$$



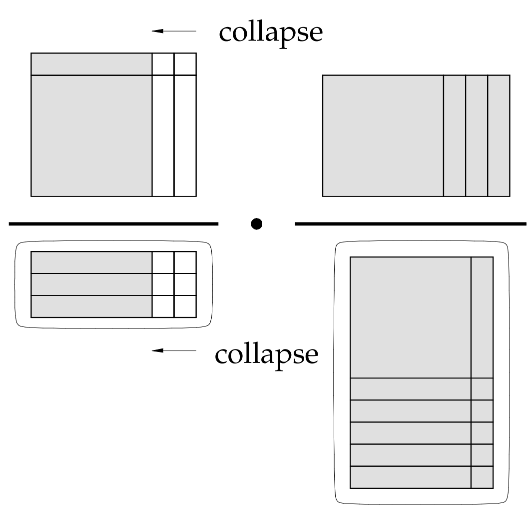
In this example we first factor (make rectangles on) the top and bottom of each fraction, and then we reduce (cancel) factors common to the numerators and denominators before multiplying to get the final result.

Here's one more example:

$$\frac{x^2 - x - 2}{3x - 6} \cdot \frac{x^2 + 3x}{x^2 + 6x + 5}$$

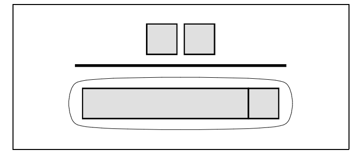


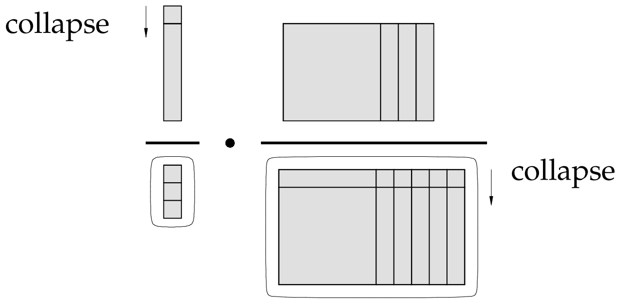
$$\frac{(x+1)\cancel{(x-2)}}{3\cancel{(x-2)}} \cdot \frac{x(x+3)}{(x+1)(x+5)}$$



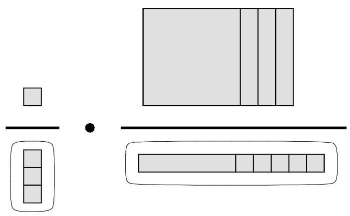
$$\frac{(x+1)}{3} \cdot \frac{x(x+3)}{(x+1)(x+5)}$$

Continuing on:

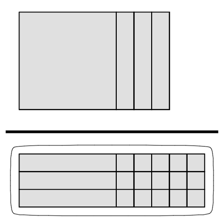




$$\frac{\cancel{(x+1)}}{3} \cdot \frac{x(x+3)}{\cancel{(x+1)}(x+5)}$$



$$\frac{1}{3} \cdot \frac{x(x+3)}{(x+5)}$$



$$\frac{x(x+3)}{3(x+5)}$$

Multiplication of rational expressions is a process of factoring the given numerators and denominators, and reducing by canceling *like factors (dimensions)* from top and bottom. Finally, the remaining factors are multiplied across the top and across the bottom. The result can be left in factored form.

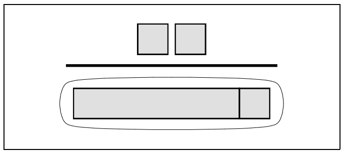
Exercises

Multiply the following rational expressions, canceling when possible and leaving answers in factored form:

1. $\frac{2}{x-2} \cdot \frac{x+2}{x+5}$

2. $\frac{x+1}{2x+3} \cdot \frac{x+3}{2x+2}$

3. $\frac{3x-6}{2x-4} \cdot \frac{2x+4}{x-3}$



4. $\dfrac{5x}{x+3} \cdot \dfrac{x-4}{x^2-4x}$

5. $\dfrac{x^2+6x}{x^2-7x+12} \cdot \dfrac{x-3}{3x+18}$

6. $\dfrac{x+7}{x^2+2x-35} \cdot \dfrac{x-5}{x^2-4x+4}$

7. $\dfrac{x+2}{x^2-x-6} \cdot \dfrac{x^2-4x+3}{x^2-5x+6}$

8. $\dfrac{x^2+5x+4}{x^2+3x-4} \cdot \dfrac{x^2-2x+1}{x^2+7x+6}$

9. $\dfrac{x^2-x-12}{3x+9} \cdot \dfrac{x^2-8x+16}{x^2-16}$

10. $\dfrac{x+2}{x-2} \cdot \dfrac{x^2+x-2}{x^2-4}$

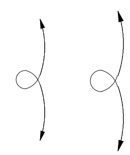
Section 6
Division

Dividing With Rational Expressions

Before attempting to divide using rational expressions, you may want to review the section on dividing with fractions in Chapter 4.

The process of division with rational expressions works in the same way as with ordinary fractions. We begin with the idea that dividing one rational expression by another is accomplished by inverting (turning over) the dividing fraction and then multiplying. When performing the multiplication we can reduce (cancel) as in the last section. For example, divide these rational expressions:

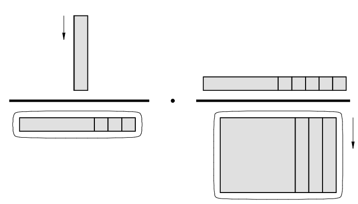
$$\frac{x}{x+3} \div \frac{x^2+3x}{x+5}$$



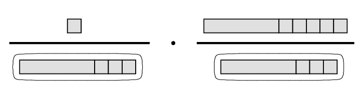
First invert the divisor and change the sign to multiply.

$$\frac{x}{x+3} \cdot \frac{x+5}{x^2+3x}$$

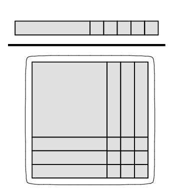
Now multiply as before:



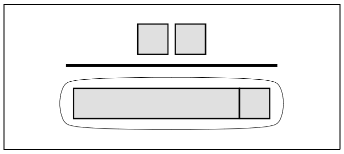
$$\frac{(x)}{x+3} \cdot \frac{x+5}{(x)(x+3)}$$



$$\frac{1}{x+3} \cdot \frac{x+5}{(x+3)}$$



$$\frac{x+5}{(x+3)(x+3)} \text{ or } \frac{x+5}{x^2+6x+9}$$



It is very important to remember in these division problems that *you cannot cancel (reduce) the given fractions until you have inverted the divisor and are ready to multiply*. Remember that

$$\frac{6}{1} \text{ divided by } \frac{1}{3} \text{ is 18, } \textit{not } 2$$

Exercises

Divide or multiply as indicated:

1. $\dfrac{x+2}{x^2-3x} \div \dfrac{2x+4}{x^2}$

2. $\dfrac{x^2-2x}{3x-9} \div \dfrac{x^2-4x+4}{x^2-5x+6}$

3. $\dfrac{x^2+7x+6}{3x+6} \cdot \dfrac{x+6}{x^2+2x+1}$

4. $\dfrac{x^2+7x+6}{3x+6} \div \dfrac{x+6}{x^2+2x+1}$

5. $\dfrac{2x+3}{2x^2+5x+3} \div \dfrac{3x^2+6x}{x^2-x-2}$

6. $\dfrac{2x-6}{x^2+x-6} \cdot \dfrac{x^2-4x+4}{x^2-3x}$

7. $\dfrac{x-2}{x+4} \div \dfrac{x^2-4}{x+1}$

8. $\dfrac{4x^2-1}{2x^2+x} \cdot \dfrac{2x+3}{2x-1}$

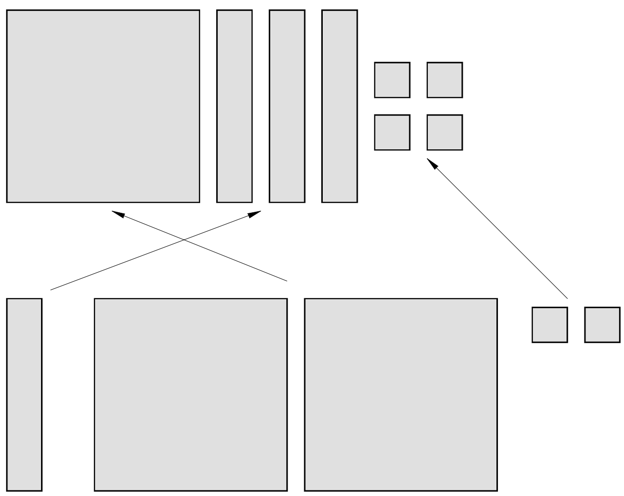
9. $\dfrac{x+3}{x-3} \div \dfrac{x^2-9}{x^2-6x+9}$

10. $\dfrac{x^2-25}{x^2-10x+25} \cdot \dfrac{x^2-2x-15}{3x-12}$

Section 7
Addition

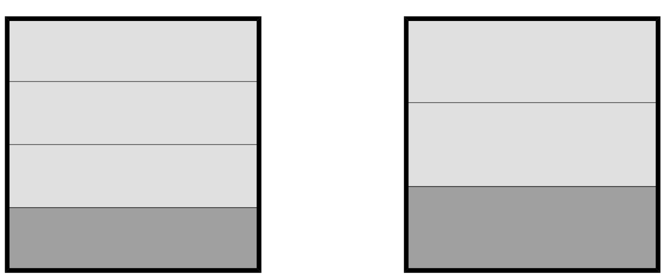
Adding and Subtracting Rational Expressions

To understand how to add rational expressions, we must briefly review the process of adding algebraic expressions and common fractions. When we combine two groups into one group (what we call adding) *we can only combine the pieces that are the same size*. For example, combine these pieces:

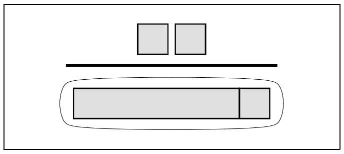


The x^2 pieces go with other x^2 pieces, x's with other x's and units with other units (which we call **combining like terms**). Pieces of different sizes must be kept in separate groups connected by a + or – sign.

When adding fractions we must go through a process to be sure that all the pieces we wish to combine are the same size. When adding two fractions, the fractions may begin as *different* size pieces. For example if we wish to add ¼ of a unit and ⅓ of a unit we begin with two pieces which are different sizes.



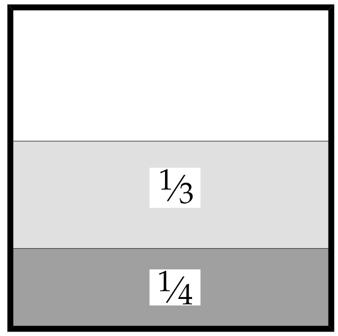
$\frac{1}{4}$ $\qquad$ $\frac{1}{3}$



We can put them side by side, but how do we write the result other than

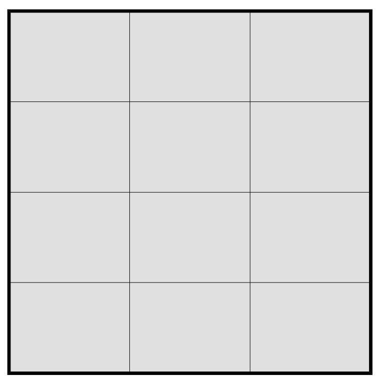
$$\frac{1}{4} + \frac{1}{3}\,?$$

Pieces of different sizes cannot combine.

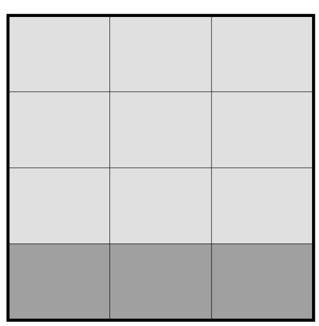


The process we go through to make both $\frac{1}{4}$ and $\frac{1}{3}$ out of pieces which are the same size is called **finding the common denominator**. Here is how it is done:

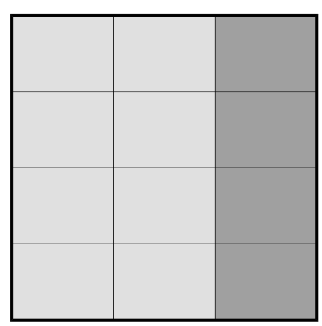
Take the unit square and divide it one direction into fourths and divide it the other direction into thirds.



Now the unit square is divided into 12 parts, and the $\frac{1}{4}$ and $\frac{1}{3}$ look like this:

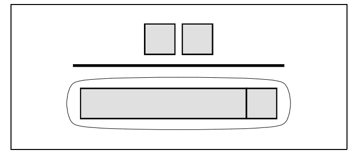


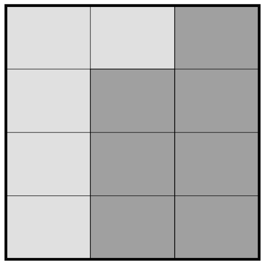
$$\frac{1}{4}$$



$$\frac{1}{3}$$

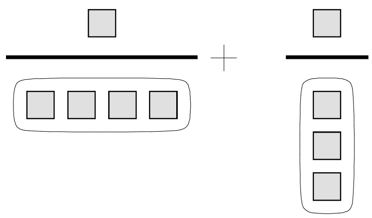
We can see that $\frac{1}{4}$ equals $\frac{3}{12}$ and $\frac{1}{3}$ equals $\frac{4}{12}$. Rearranging the shaded rectangles will allow us to show that the total is $\frac{7}{12}$:



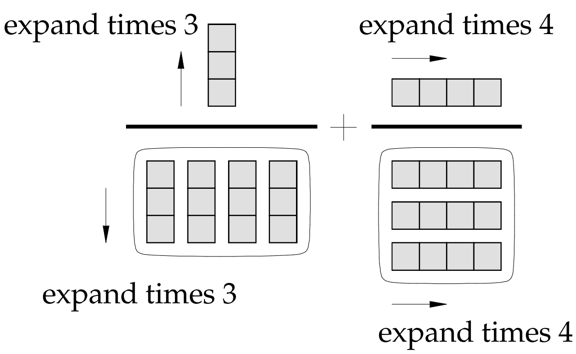


$$\frac{1}{4} = \frac{3}{12} \quad \text{and} \quad \frac{1}{3} = \frac{4}{12}$$

If we use the new method of representing rational expressions, this would be pictured as follows:

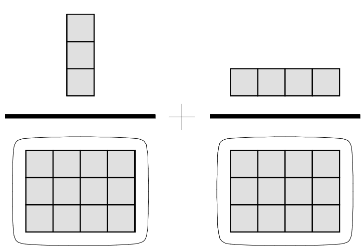


$$\frac{1}{4} + \frac{1}{3}$$



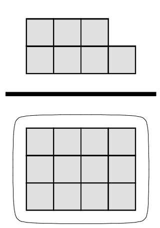
Now we expand the top and bottom of both fractions so that their denominators become rectangles of the same size. (This is the reverse process of collapsing or reducing.)

$$\frac{3 \cdot 1}{3 \cdot 4} + \frac{4 \cdot 1}{4 \cdot 3}$$

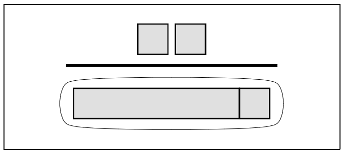


$$\frac{3}{12} + \frac{4}{12}$$

When the units (denominators) are the same, the pieces can be combined (added together).



$$\frac{7}{12}$$



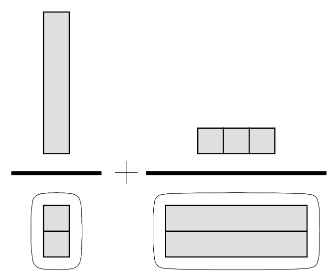
Here we see that, rather than collapsing rectangles which have like dimensions, we have expanded rectangles to give them like dimensions. We are making the units the same size (same denominator) so we can combine (add) the numerators together. If we begin with two denominators which share no factors (like 4 and 3), then the final rectangle required for the denominator will have each of the original denominators as one dimension ($4 \cdot 3 = 12$).

In order to add fractions together, they must both have the same denominator (the same size pieces). The same requirement is true for adding positive and negative fractions, or subtracting fractions.

In order to add fractions together, they must both have the same denominator (the same size pieces).

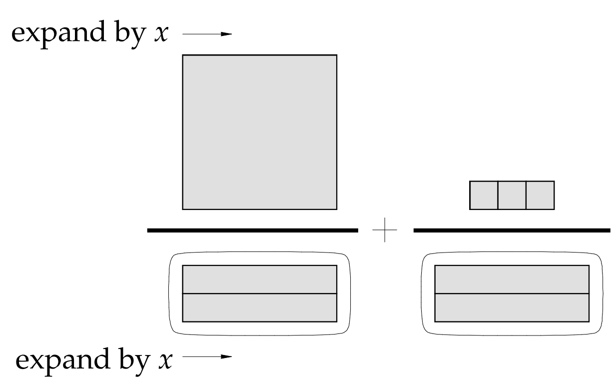
Since rational expressions are simply fractions having unknown terms, this same requirement of having common denominators also holds when we wish to add or subtract rational expressions. To meet this requirement, we often must expand the dimensions of the rectangles used in one or both fractions to create denominator rectangles which are the same for both fractions being added. For example, let's add the following:

$$\frac{x}{2} + \frac{3}{2x}$$

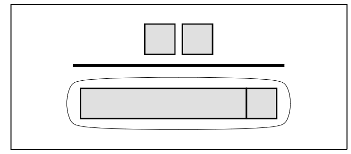


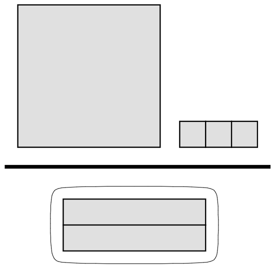
The first denominator rectangle has dimensions 2 by 1, while the other denominator rectangle has dimensions 2 by x. To make the dimensions of the two denominators the same, we must expand the first denominator (and also the first numerator) to have a width of x.

$$\frac{x \cdot x}{x \cdot 2} + \frac{3}{2x} = \frac{x^2}{2x} + \frac{3}{2x}$$



Now that the denominators are the same, we can put both numerators over the same denominator.



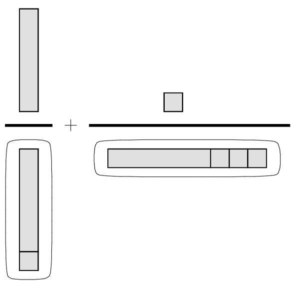


$$\frac{x^2+3}{2x}$$

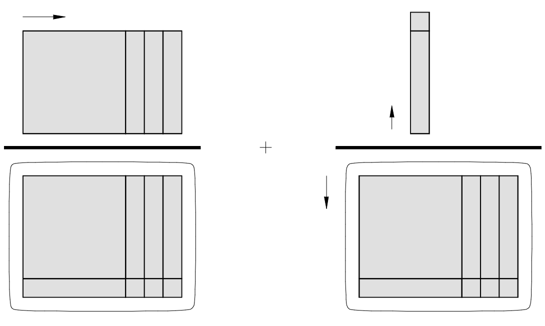
Since the numerator of this expression won't factor, this is our final result.

You will notice that adding two rational expressions means that they get combined over the same denominator (into one fraction), but it doesn't necessarily mean that the numerator simplifies greatly in form.

Here's another example:

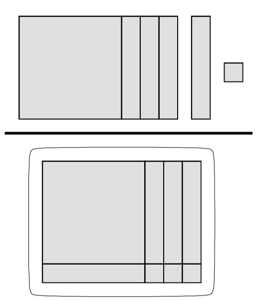


$$\frac{x}{x+1}+\frac{1}{x+3}$$



Expand to give like denominators:

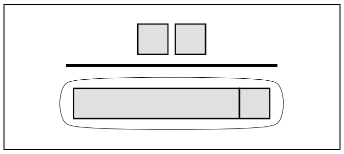
$$\frac{x^2+3x}{(x+1)(x+3)}+\frac{x+1}{(x+1)(x+3)}$$



Combine into one fraction:

$$\frac{x^2+4x+1}{(x+1)(x+3)}$$

Since the numerator cannot be factored, this fraction cannot be reduced.

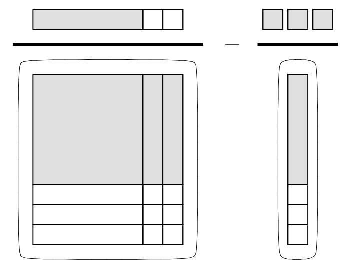


We will give one example of a **subtraction** of rational expressions. Just like with common fractions, a negative sign in front of a rational expression will flip the chips (change the sign) of the numerator, but leave the denominator (the size of a unit) unchanged. For example;

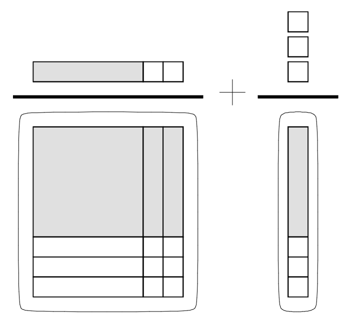
$$-\left(\frac{3}{7}\right) = \frac{^-3}{7}$$

Here is an example of subtracting rational expressions:

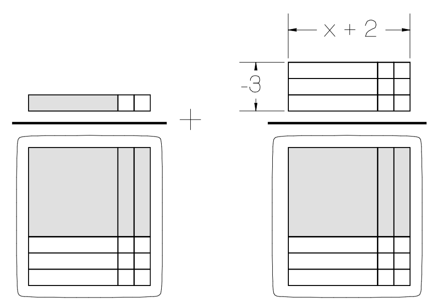
$$\frac{x-2}{x^2-x-6} - \frac{3}{x-3}$$



$$\frac{x-2}{(x+2)(x-3)} + \frac{^-3}{x-3}$$

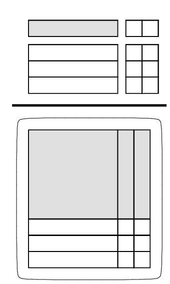


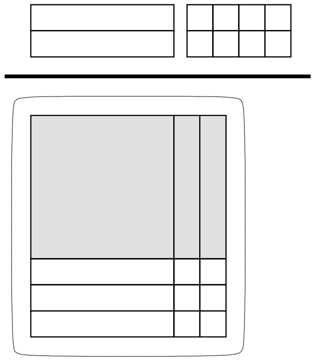
$$\frac{x-2}{(x+2)(x-3)} + \frac{^-3(x+2)}{(x-3)(x+2)}$$

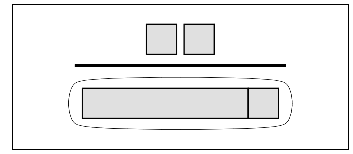


$$\frac{x-2}{(x+2)(x-3)} + \frac{^-3x-6}{(x-3)(x+2)}$$

$$\frac{x-2+{}^-3x-6}{(x+2)(x-3)}$$







$$\frac{^{-}2x-8}{(x+2)(x-3)} = \frac{^{-}2(x+4)}{(x+2)(x-3)}$$

Exercises

Add or subtract as indicated; reduce your answers:

1. $\frac{2}{3}+\frac{1}{4}$

2. $\frac{3}{5}-\frac{1}{2}$

3. $\frac{3}{7}-\frac{1}{3}$

4. $\frac{5}{6}+\frac{1}{2}$

5. $\frac{3}{x}+\frac{2}{3}$

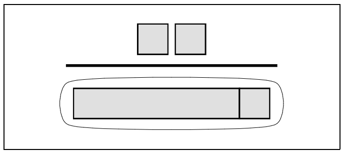
6. $\frac{2}{x+1}-\frac{3}{x}$

7. $\frac{3x}{x-2}+\frac{1}{x+1}$

8. $\frac{x}{8}-\frac{3}{5}$

9. $\frac{3x}{x+3}-\frac{5}{6}$

10. $\frac{x-1}{x-5}+\frac{7}{x+2}$



11. $\frac{2}{2x+3} - \frac{x+1}{x-1}$

12. $\frac{2x-4}{x^2+6x} + \frac{3}{x+6}$

13. $\frac{3x-2}{x^2-4x+4} - \frac{2}{x-2}$

14. $\frac{x+1}{x^2-x-6} + \frac{x}{x-3}$

15. $\frac{3x+1}{x^2+3x-10} + \frac{4}{x+5}$

16. $\frac{x+3}{x^2+x-6} - \frac{x-1}{x-2}$

Section 8
Summary

Summary of the Steps

A summary of the processes involved in multiplication, division, addition and subtraction of rational expressions is as follows:

Multiplication

- Factor the numerator and denominator of the expressions to be multiplied.
- Any factors which appear in both a numerator and a denominator cancel out.
- After canceling, multiply the remaining numerators together and the remaining denominators together to give the numerator and the denominator respectively of the resulting fraction.

Division

- Invert the divisor (the expression being divided by)
- Proceed to multiply the resulting expressions.

Addition

- Expand one or both fractions (top and bottom) to give both fractions the same (common) denominator.
- Add the numerators together and leave the result over this common denominator as a single fraction.
- Factor the numerator and denominator and check to see if they have any dimensions in common which could reduce (collapse) out.

Subtraction

- In the fraction being subtracted, take the opposite (negative) of the numerator, and proceed by adding the fractions together.

Exercises

Perform the indicated operations; reduce your answers:

1. $\frac{3}{x} + \frac{x}{8}$

2. $\frac{x}{x+2} \cdot \frac{2x+1}{3x}$

3. $\frac{x^2+5x+6}{3x+15} \div \frac{x+2}{x+5}$

4. $\frac{x+1}{x^2-4x} - \frac{2}{x}$

5. $\frac{x+2}{x^2-4} + \frac{x}{x+1}$

6. $\frac{x-2}{x^2+4x+3} \div \frac{x+1}{x^2-x-2}$

7. $\frac{x-2}{x^2+4x+3} \cdot \frac{x+1}{x^2-x-2}$

8. $\frac{x}{x-2} - \frac{2}{x}$

9. $\frac{x^2+6x+9}{x^2-x-2} \cdot \frac{x^2+3x+2}{x^2+5x+6}$

10. $\frac{x-2}{x+3} - \frac{3}{x-1}$

11. $\frac{x^2+3x}{x^2-4x+4} \div \frac{x^2+4x+3}{x^2-3x+2}$

12. $\frac{x+3}{x} + \frac{2x}{x-5}$