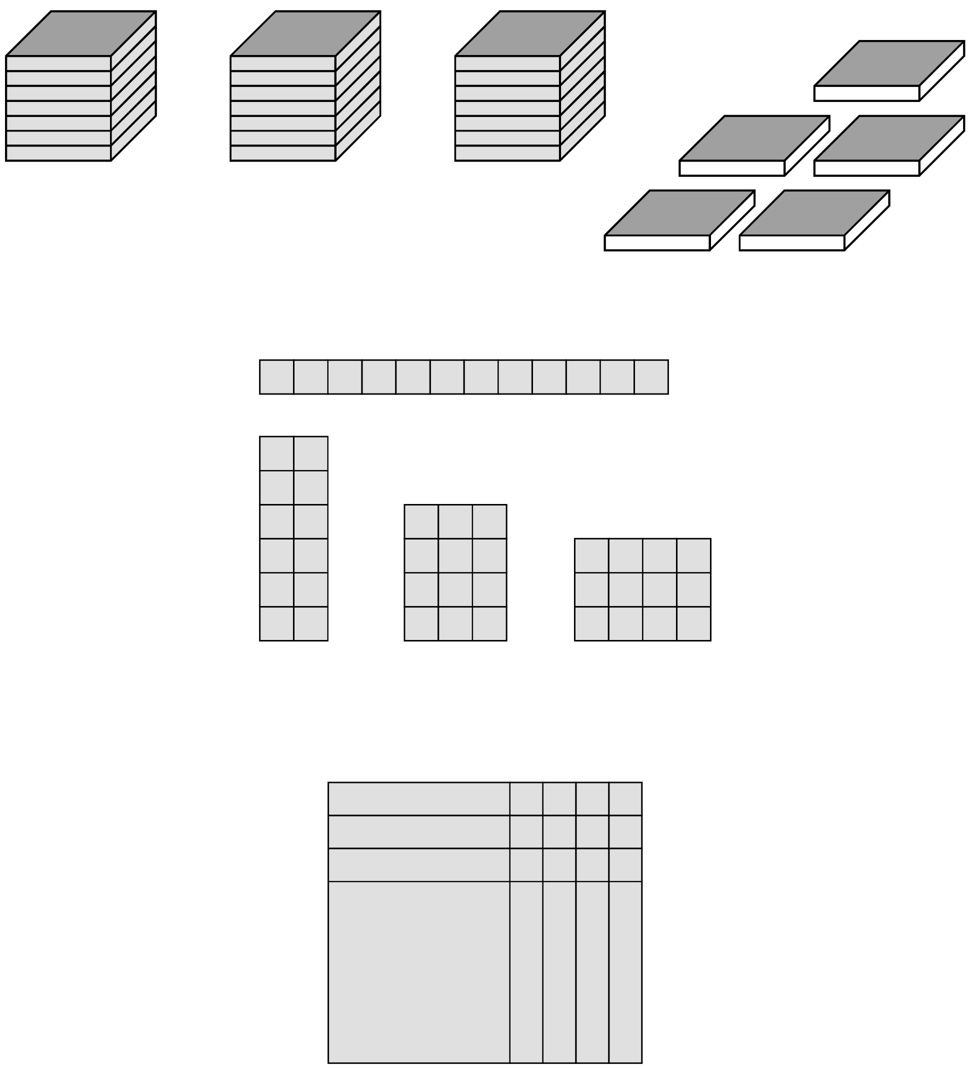
Chapter 1

Introduction



Learning By Discovery

Purpose of this chapter

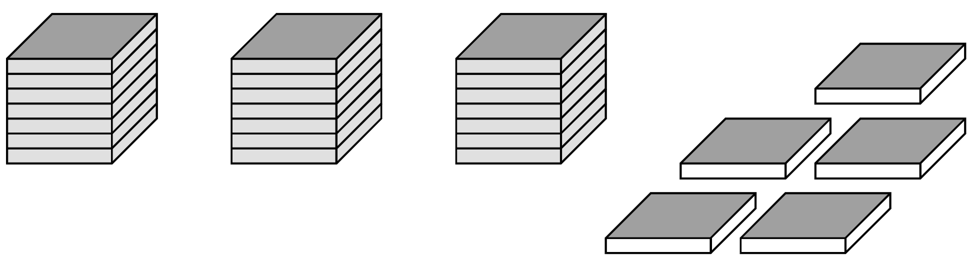
This chapter is a short introduction to the methods of the book. Three demonstrations will be presented. Each of these concepts will be covered in detail in a later chapter, so it is not important to try to memorize or practice these examples; instead you need only follow the demonstrations and enjoy the challenge of solving the problems. Your job is to understand, not to obey.

For the demonstrations, you will need the cardboard chips and a pencil and paper. As in the rest of the book, you will learn more if you follow along with the text by doing the work instead of merely reading or watching.

Demonstration 1: Solving Equations

Count out 26 of the small cardboard chips. Remove 5 chips and set them aside. Take the remaining chips and stack them up in three equal piles. *Do not count the piles; you can tell if they are equal by feeling that each pile is the same height.*

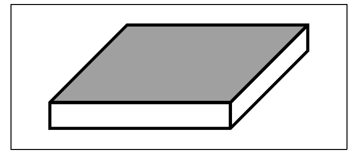
Here is what you should have:

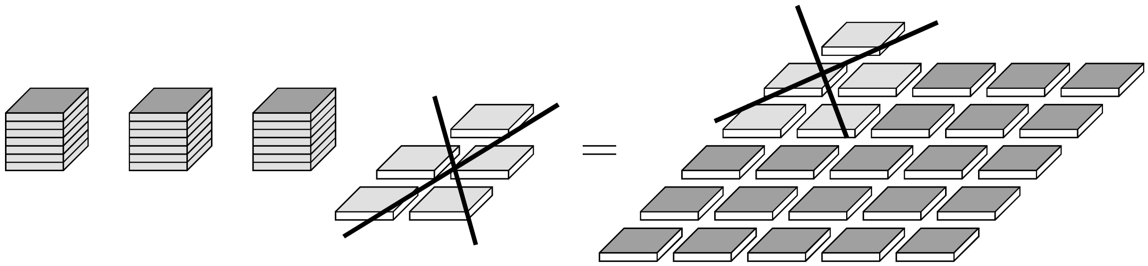


Place the three stacks and the five extra chips on a piece of paper and write an equals sign and the number 26. You have now made a statement that

$$3 \text{ stacks} + 5 = 26$$

Now, *without counting the stacks*, can you figure out how many chips are in each stack? It is not usually difficult; most people do something like this:





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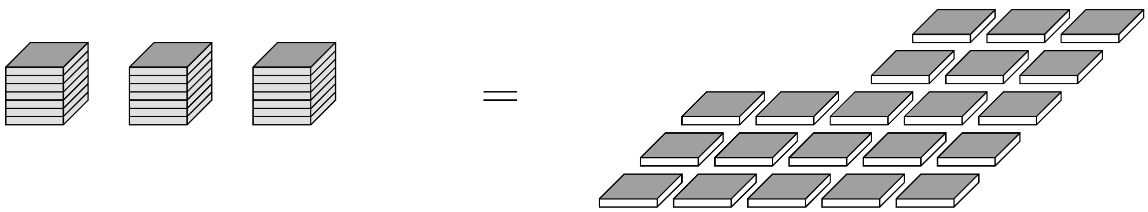

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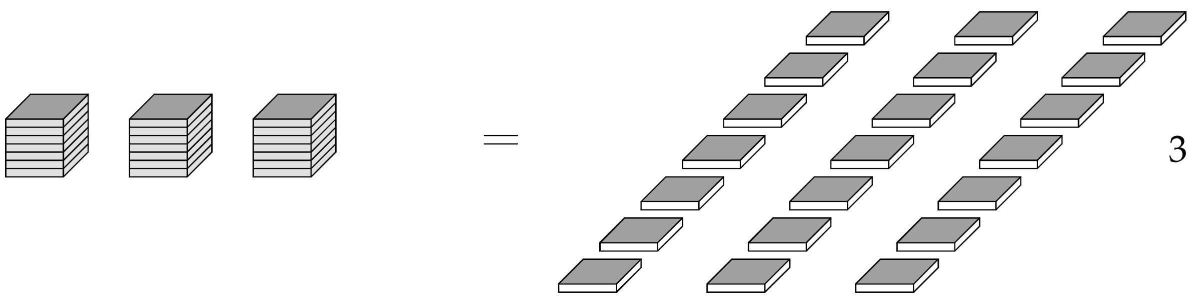
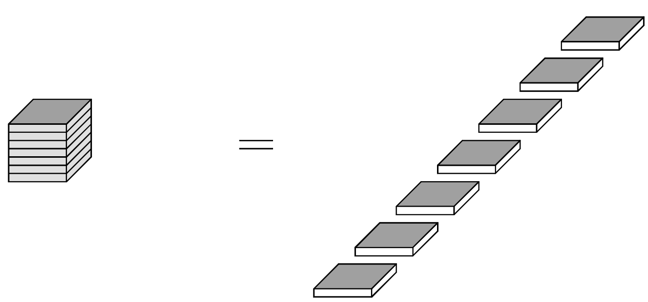


$3 \cdot 7$



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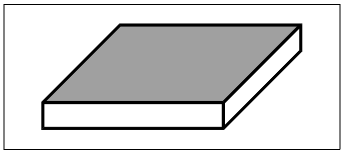
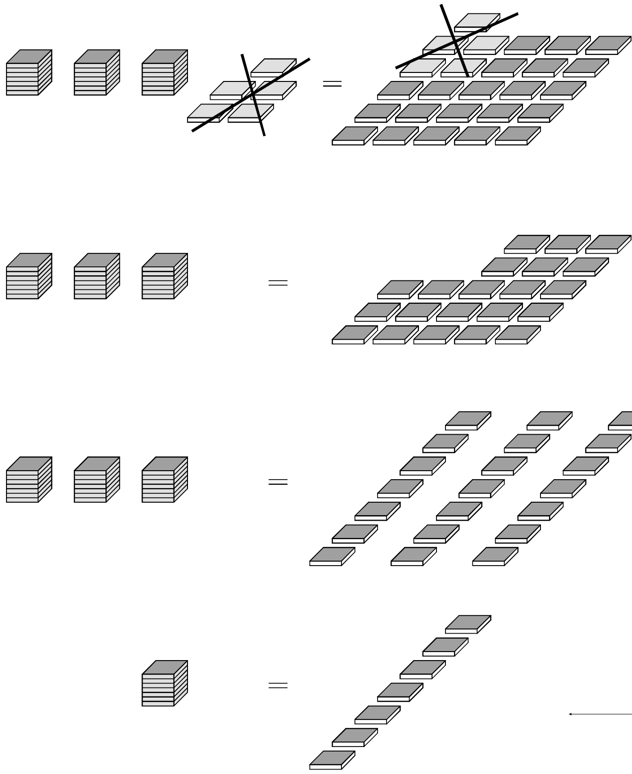
If the three stacks plus five are a total of 26 chips, then the three stacks alone must be 26 minus 5 or 21. If three equal stacks total 21, then each must be 21 divided by 3, or 7. Count a stack, and you will find that you were correct.



With algebra symbols, this is how you would do it:



$$\begin{aligned} 3x + 5 &= 26 \\ -5 \quad & \quad -5 \end{aligned}$$

$$3x = 21$$

$$\frac{3x}{3} = \frac{21}{3}$$

$$x = 7$$

As you can see, the ***x*** stands for the number of chips in a stack and ***3x*** stands for the number in three stacks. *In this book, the symbols you use will stand for something real; the algebra techniques will generally be shown as a movement of chips rather than just a manipulation of symbols.*

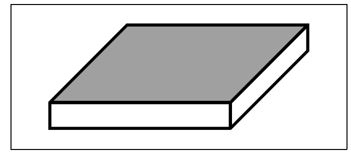
You have just solved a **linear equation in one variable**. As you progress through this book, you will find that most of algebra is this easy; you may also find, as you did here, that you know many of the concepts already.

Demonstration 2: Factoring

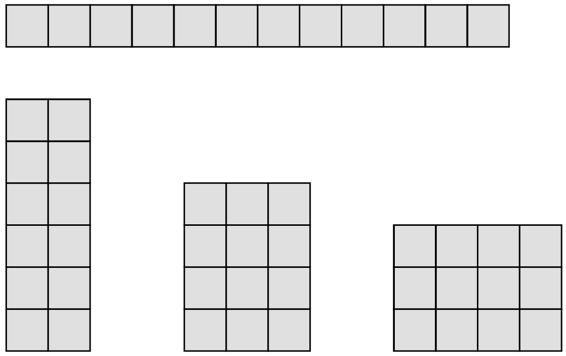
The mysterious art of factoring is usually thought to require lengthy practice and repetition. Here you will do it painlessly in a few minutes.

For this exercise, you will need:

- **1 large square**
- **7 long bars**
- **12 small squares**



First we will do a preparatory exercise. Take the 12 small squares and arrange them into a rectangle. There are several possibilities:



We call this **factoring**. When we take 12 and arrange it as 2 groups of 6, we say that

$$12 = 2 \cdot 6$$

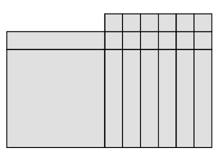
The other possibilities are:

$$12 = 3 \cdot 4$$
$$12 = 1 \cdot 12$$

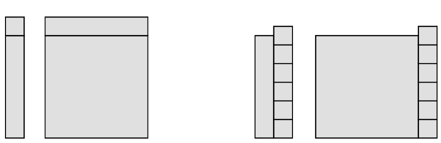
Factoring is making rectangles.

Now we will do the main exercise using all of the chips listed above. Your job is to rearrange these chips into a rectangle. Here are the rules of the game:

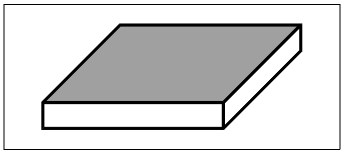
- **You must make a smooth rectangle. No holes or projecting chips are allowed. A square is considered to be a type of rectangle.**

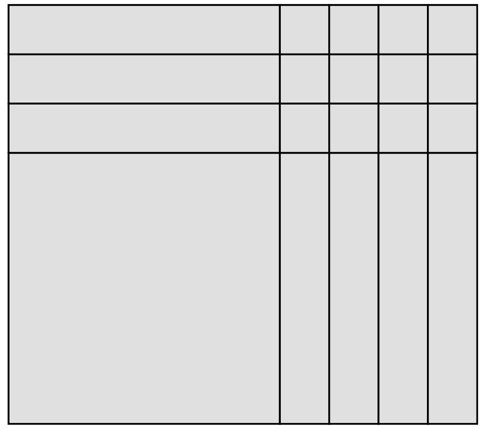
No 

- **The small squares will only match with the end of the bars. They will not fit along the side of the large square or on the long side of the bars.**

Yes  No

Try it now. If you get stuck, keep moving the pieces around until you see the answer. Our solution is on the following page (yours may be slightly different):

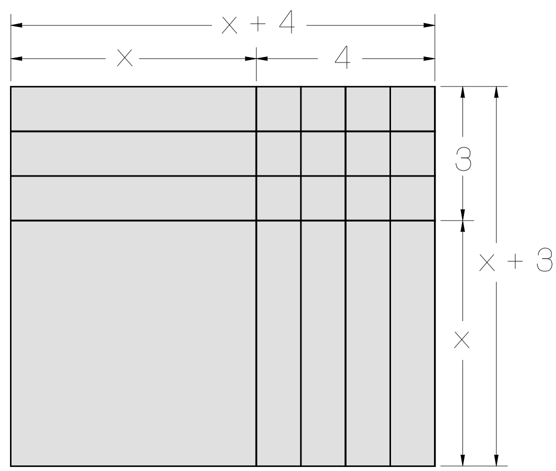




However your rectangle is formed, it will have a bar and three chips along one edge, and a bar and four chips along the other edge.

In the language of algebra, the small squares stand for **1**, the bars stand for x, the large squares stand for x^2 (read as "x squared"). Because the finished rectangle has a length of one bar plus three chips and a width of one bar plus four chips, we say that:

$$x^2 + 7x + 12 = (x + 4)\cdot(x + 3)$$



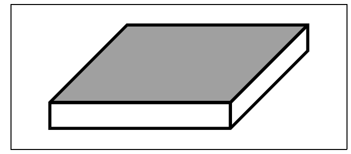
You have just **factored a quadratic expression**.

Demonstration 3: Dividing Fractions

Without writing anything down, can you quickly answer this question:

$$8 \div \frac{1}{4} = ?$$

Do not work out the answer using any rules.



Typical answers are 2, 12, 32, and "I don't know." If you can recall the rules of arithmetic, you would probably do it this way:

$$8 \div \frac{1}{4} = 8 \cdot \frac{4}{1} = \frac{32}{1} = 32$$

Invert (Why?) Multiply

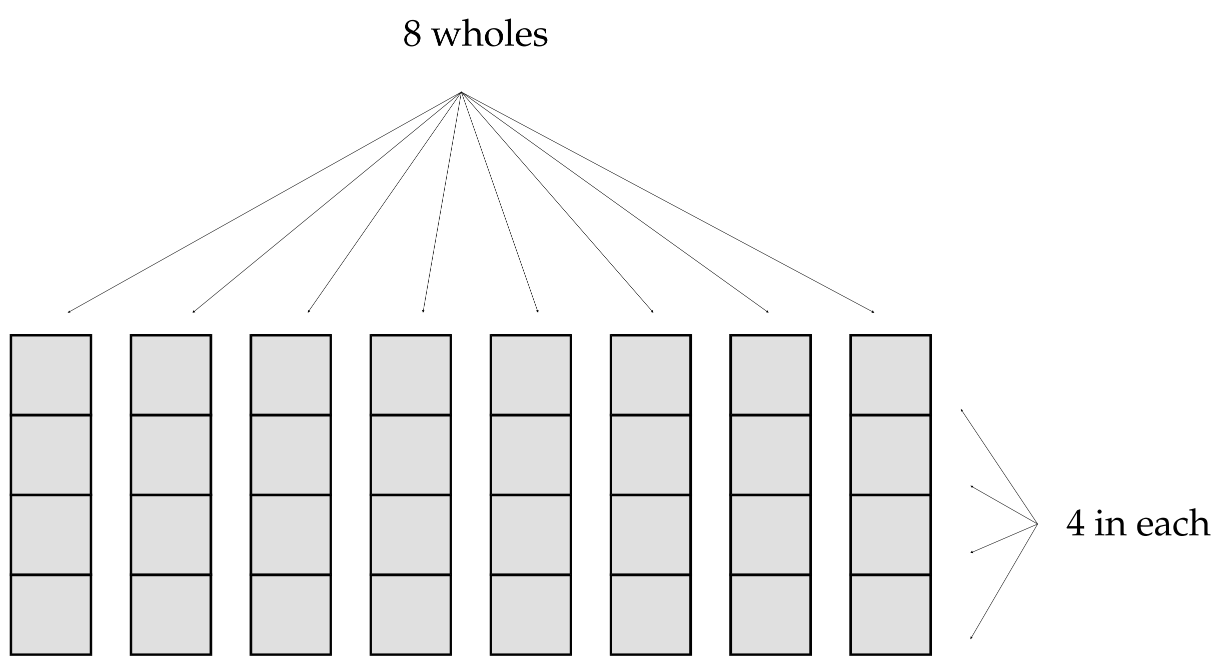
What does this mean? Let's pose this question another way:

How many quarters are in $8.00?

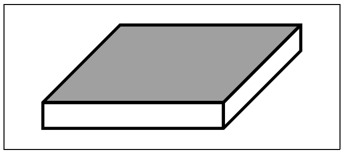
Since there are 4 quarters in 1 dollar, there are $4 \cdot 8$ or 32 quarters in 8 dollars.

Was this second problem much easier than the first? Yes, because it meant something very real. In fact, most people are able to do it even if they do not remember rules about dividing fractions.

Now we will do the problem with chips. If we decide that 4 chips together are one whole unit, then each chip is ¼. Set up 8 whole units like this:



You can see that there are 8 groups of 4 or $8 \cdot 4 = 32$ quarters.



Summary

Here are some of the important lessons of this chapter:

- **The symbols of arithmetic and algebra can stand for real objects.**
- **We already know many of the concepts of algebra.**
- **The best way to learn algebra is to understand the meaning of the symbols, techniques, and properties. Understanding algebra is more enjoyable and more efficient than memorizing a list of rules.**

Exercises

Use the chips to solve these problems:

Solving equations: Set up stacks and determine how many chips are in a stack. Remember that x is a stack.

1. $4x + 2 = 26$ (Use 26 chips)

2. $2x + 9 = 19$

3. $7x + 6 = 34$

Factoring: Make the chips into a rectangle.

4. $x^2 + 8x + 12$

5. $x^2 + 8x + 15$

6. $2x^2 + 5x + 2$ ($2x^2$ means two large squares)

Dividing fractions: Show the answer with the chips.

7. $6 \div \frac{1}{3}$

8. $4 \div \frac{1}{2}$

9. $4 \div \frac{2}{3}$

10. $12 \div \frac{3}{4}$

11. $3 \div \frac{1}{3}$

12. $2 \div \frac{2}{3}$