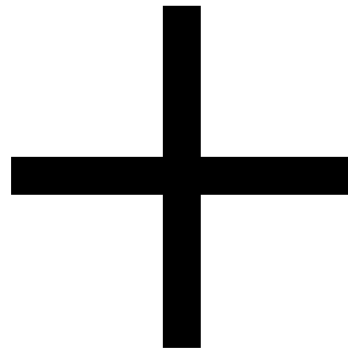




# Appendices



# Appendix 1

## Division of Fractions: Part 2

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### Dividing a Fraction by a Number

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All of the examples in the FRACTIONS chapter covered situations of dividing an integer by a fraction. There are other cases such as:

$$\frac{1}{2} \div 6 \quad \text{or} \quad \frac{1}{3} \div \frac{2}{3}$$

The visual methods used before will also work for these cases, but the pictures will be a little more complex. Consider

$$\frac{1}{2} \div 6 = \frac{1}{2} \div \frac{6}{1}$$

Our rules tell us:

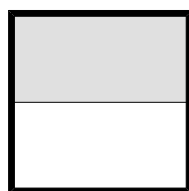
$$\frac{1}{2} \div \frac{6}{1} = \frac{1}{2} \cdot \frac{1}{6} = \frac{1 \cdot 1}{2 \cdot 6} = \frac{1}{12}$$

What does this problem look like? There are several ways to illustrate it:

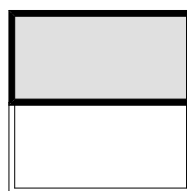
- **Divide  $\frac{1}{2}$  into 6 equal parts. How large is each part?**
- **How many 6's can we make out of  $\frac{1}{2}$ ?**
- **Make a rectangle out of  $\frac{1}{2}$  unit. Arrange it with 6 in one direction. How long is the other direction?**

All of these are equivalent and of course give the same result. Because each method is useful in certain situations, you should work on understanding all of them.

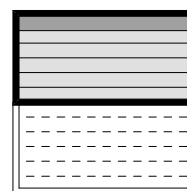
First, let's divide  $\frac{1}{2}$  into 6 equal parts. The answer (each equal part) is  $\frac{1}{12}$ :



1 whole

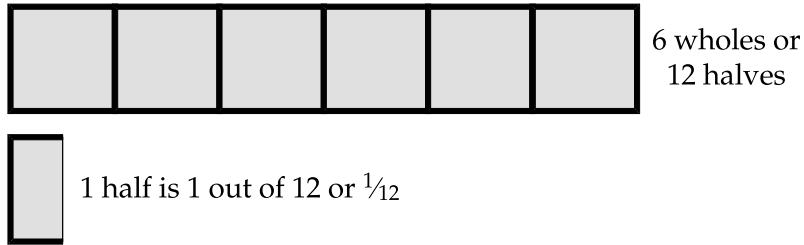
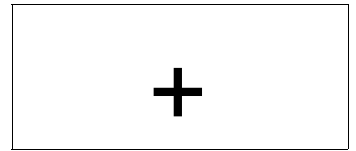


$\frac{1}{2}$

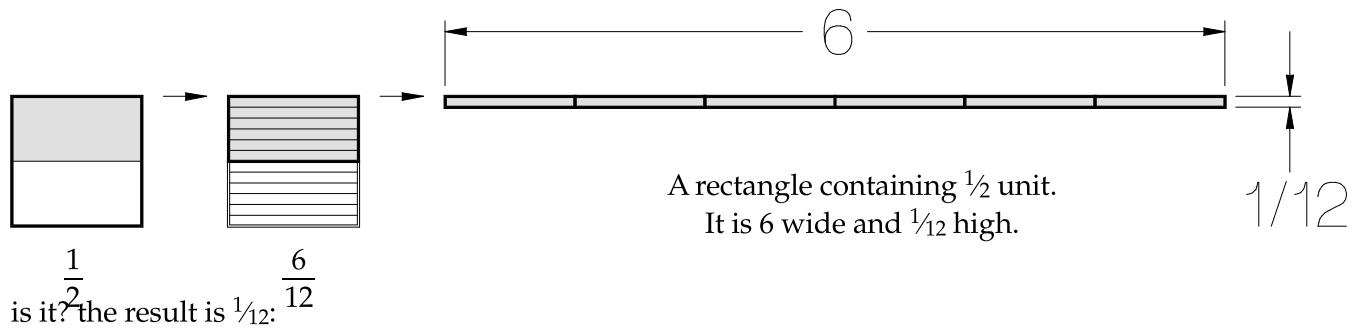


$\frac{1}{2}$  divided into  
6 groups of  $\frac{1}{12}$ .

Now, let's see how many 6's you can make out of  $\frac{1}{2}$ . If you compare 6 and  $\frac{1}{2}$ , it is clear that  $\frac{1}{2}$  is  $\frac{1}{12}$  of 6. You can make  $\frac{1}{12}$  of a 6 from  $\frac{1}{2}$ :

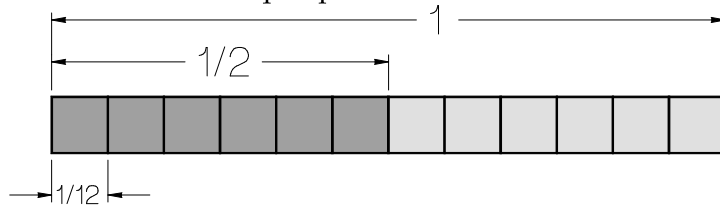


Finally, let's take  $\frac{1}{2}$  and arrange it into a rectangle that is 6 wide. How high

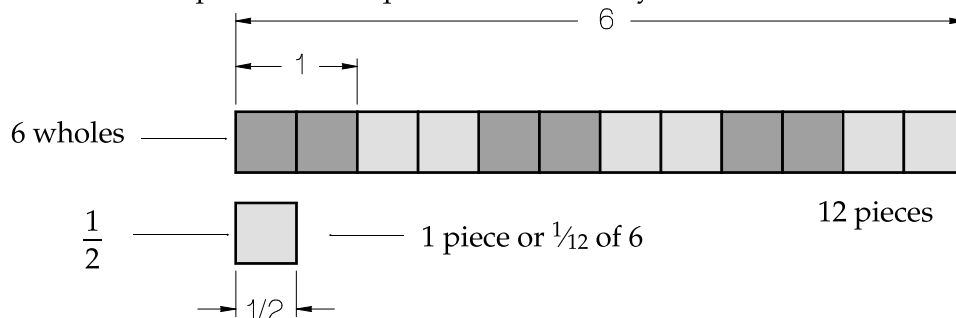


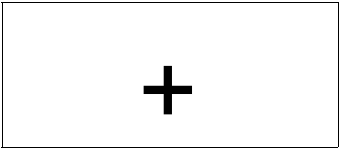
### Using the Chips

You can also use the chips to represent these division examples. Remember that each chip takes on different dimensions in each problem. First, here is a picture of  $\frac{1}{2}$  divided into 6 equal pieces:

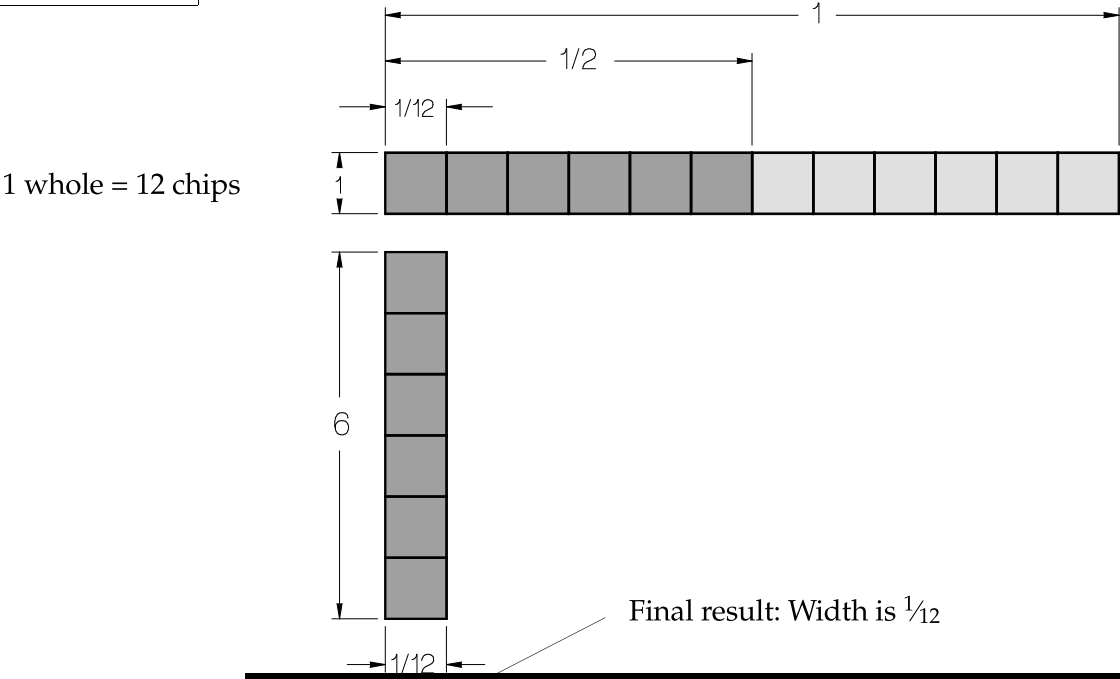


Next, here is a picture of the question "How many 6's in  $\frac{1}{2}$ ?"





Finally, here is the rectangle approach where we arrange  $\frac{1}{2}$  into a rectangle that is 6 on one side and  $\frac{1}{12}$  on the other:




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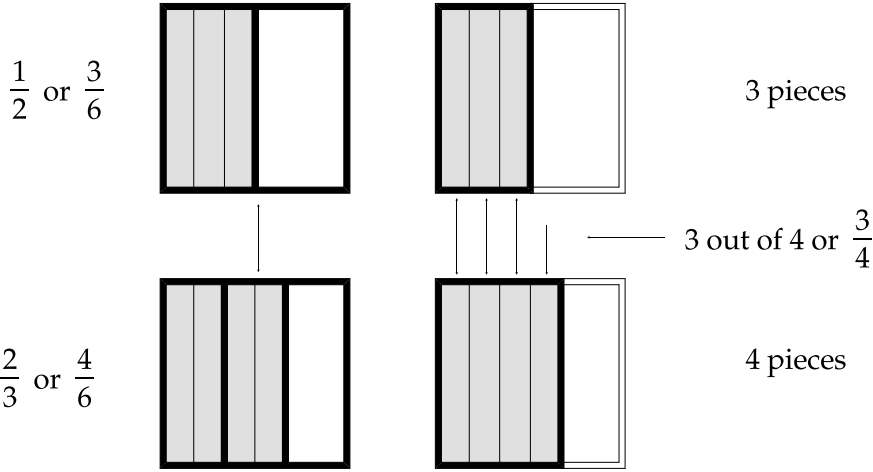
**Dividing a Fraction by a Fraction**

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We will conclude with two pictures of

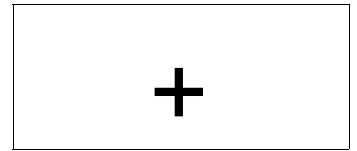
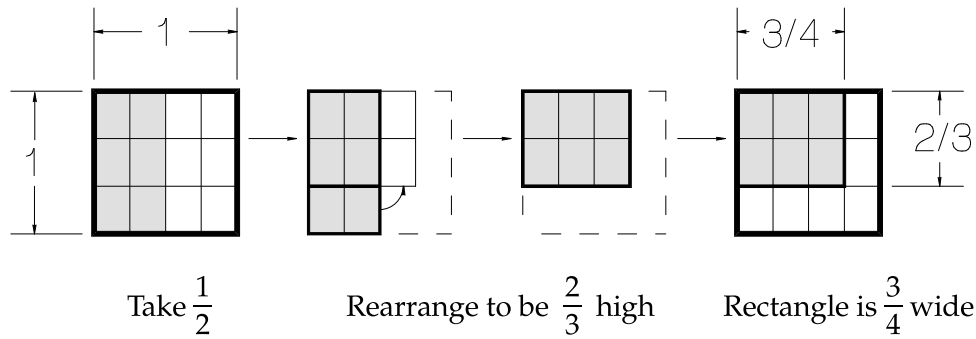
$$\frac{1}{2} \div \frac{2}{3}$$

If we think of the question as “How many  $\frac{2}{3}$ 's can we make from  $\frac{1}{2}$ ,” then we compare  $\frac{1}{2}$  to  $\frac{2}{3}$  and use one-sixth as a common unit.



We see 3 units (sixths) in  $\frac{1}{2}$  as opposed to 4 units in  $\frac{2}{3}$ , so our result is that there are 3 out of 4 or  $\frac{3}{4}$ .

A second method is to build a rectangle from  $\frac{1}{2}$  that is  $\frac{2}{3}$  high. The width ( $\frac{3}{4}$ ) is our result:



The rule of “invert and multiply” still works:

$$\frac{1}{2} \div \frac{2}{3} = \frac{1}{2} \cdot \frac{3}{2} = \frac{1 \cdot 3}{2 \cdot 2} = \frac{3}{4}$$

Now we see where the rule comes from. To summarize our method:

### Dividing Two Fractions

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} = \frac{a \cdot d}{b \cdot c} = \frac{ad}{bc}$$

### Exercises

Use pictures and symbols to solve these problems. If chips are used, remember that each chip represents a different area in each problem.

1.  $\frac{3}{4} \div \frac{2}{3}$
2.  $\frac{1}{2} \div 4$
3.  $\frac{2}{3} \div 4$
4.  $\frac{1}{3} \div \frac{2}{3}$
5.  $\frac{2}{3} \div \frac{1}{3}$

## Appendix 2

### Mixed Numbers

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#### Multiplying Mixed Numbers

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Fractions greater than one are sometimes called **mixed numbers** when written like this:

$$1\frac{1}{2}$$

and **improper fractions** when written like this:

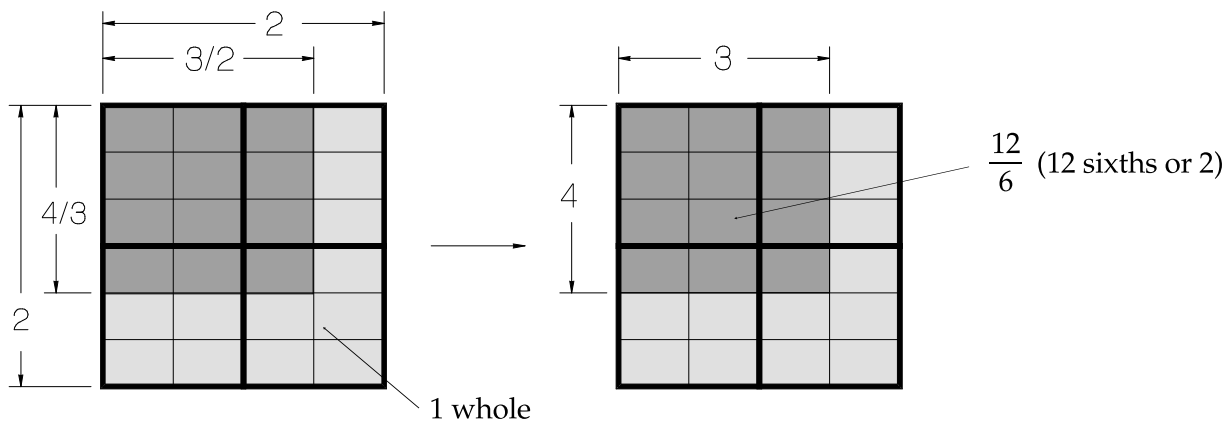
$$\frac{3}{2}$$

In most uses of algebra notation, we will be representing these numbers as one fraction rather than as a mixed number. *In these examples, always change mixed numbers into one fraction.*

Our examples so far have been limited to integers and fractions less than one. It is not difficult to extend our system to include more complicated fractions. Because of the number of chips involved, you may prefer to draw the pictures rather than use the chips. Consider the problem:

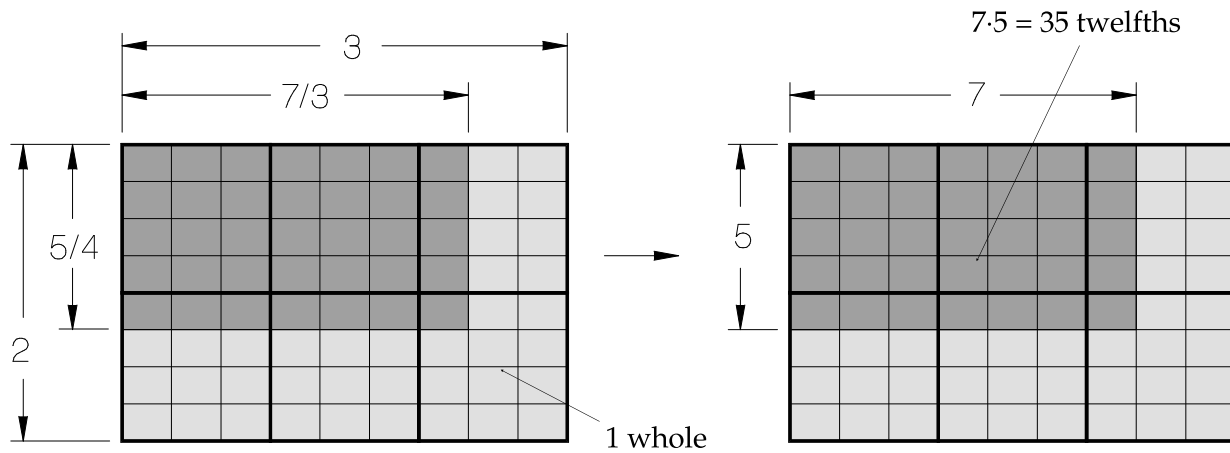
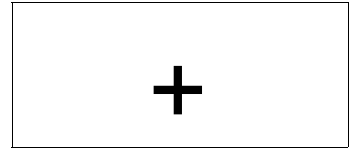
$$\frac{3}{2} \cdot \frac{4}{3}$$

This is a rectangle that is  $\frac{3}{2}$  long and  $\frac{4}{3}$  high. Because we have halves in one direction and thirds in another, the individual pieces are sixths ( $2 \cdot 3$ ). Because the resulting rectangle has four pieces in one direction and three in the other, we have  $4 \cdot 3$  or 12 pieces. The result is 12 sixths ( $\frac{12}{6}$  or 2):



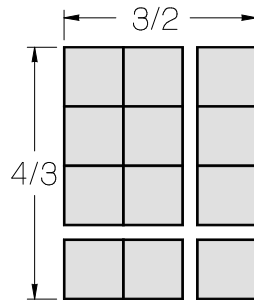
Here is another example:

$$\frac{7}{3} \cdot \frac{5}{4}$$



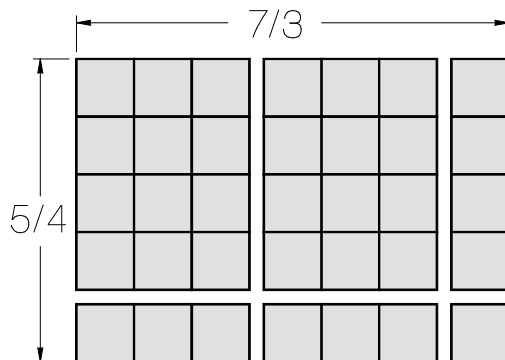
### Using Chips for Multiplication

To use the chips, remember that each chip will represent a different area in each problem. Here is  $\frac{3}{2} \cdot \frac{4}{3}$ :

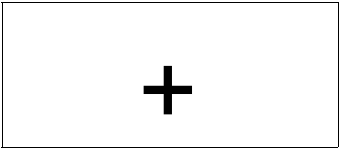


Chips are  $\frac{1}{6}$  ( $\frac{1}{2}$  by  $\frac{1}{3}$ ).  
We have  $3 \cdot 4 = 12$  chips.  
The result is  $\frac{12}{6}$  or 2.

Here is  $\frac{7}{3} \cdot \frac{5}{4}$ :



Chips are  $\frac{1}{12}$  ( $\frac{1}{4}$  by  $\frac{1}{3}$ ).  
We have  $7 \cdot 5 = 35$  chips.  
The result is  $\frac{35}{12}$ .



## Division of Mixed Numbers

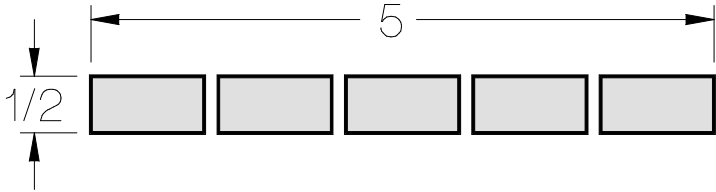
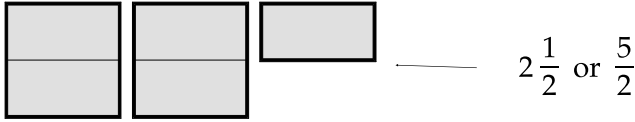
Like multiplication, division is not fundamentally different when done with fractions greater than 1. The meaning remains the same:

$$\frac{5}{2} \div \frac{1}{2} \text{ means "How many one-halves in five-halves?"}$$

or

"Take  $\frac{5}{2}$  and build a rectangle that is  $\frac{1}{2}$  high. How wide is it?"

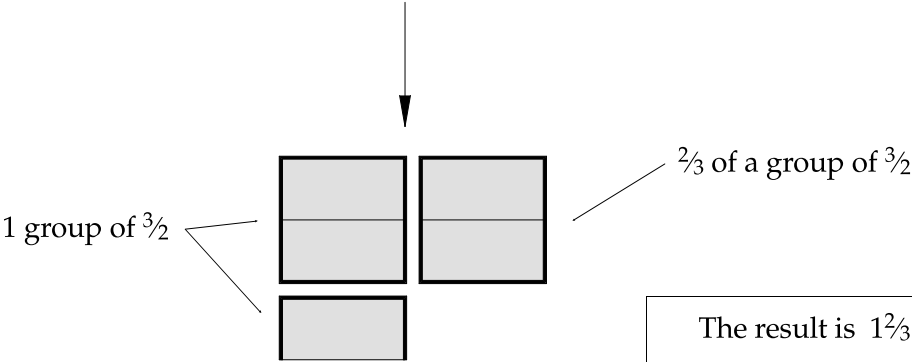
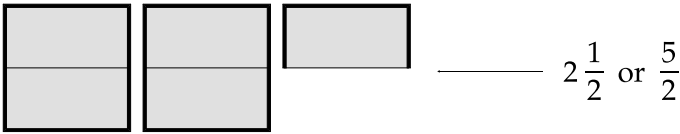
Here is an illustration:



Build a rectangle  $\frac{1}{2}$  high.  
It is 5 wide. The result is 5.

Here is a second example:

$$\frac{5}{2} \div \frac{3}{2} \text{ means "how many groups of } \frac{3}{2} \text{ are in } \frac{5}{2} \text{?"}$$

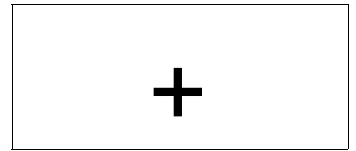


The result is  $1\frac{2}{3}$



## Exercises

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Draw pictures or use chips to solve the following problems. Write out your work in symbols as well.

1.  $\frac{3}{2} \cdot \frac{5}{4}$

2.  $\frac{7}{3} \cdot \frac{4}{3}$

3.  $\frac{7}{4} \cdot \frac{7}{3}$

4.  $\frac{3}{2} \div \frac{1}{4}$

5.  $\frac{1}{4} \div \frac{3}{2}$

6.  $\frac{1}{4} \div \frac{5}{2}$

7.  $\frac{3}{4} \div \frac{5}{2}$

8.  $1\frac{1}{4} \div \frac{5}{2}$

9.  $1\frac{1}{4} \div 1\frac{1}{2}$

10.  $1\frac{1}{4} \div 1\frac{1}{4}$