Appendix

+

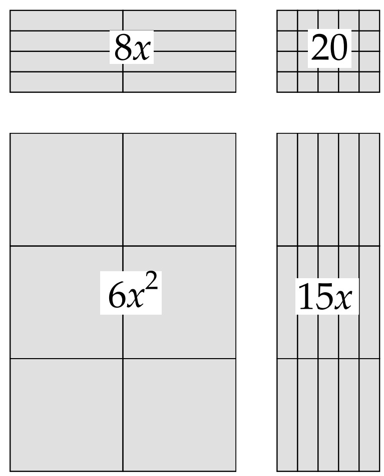
Appendix
Factoring By Grouping

The Shortcut Method

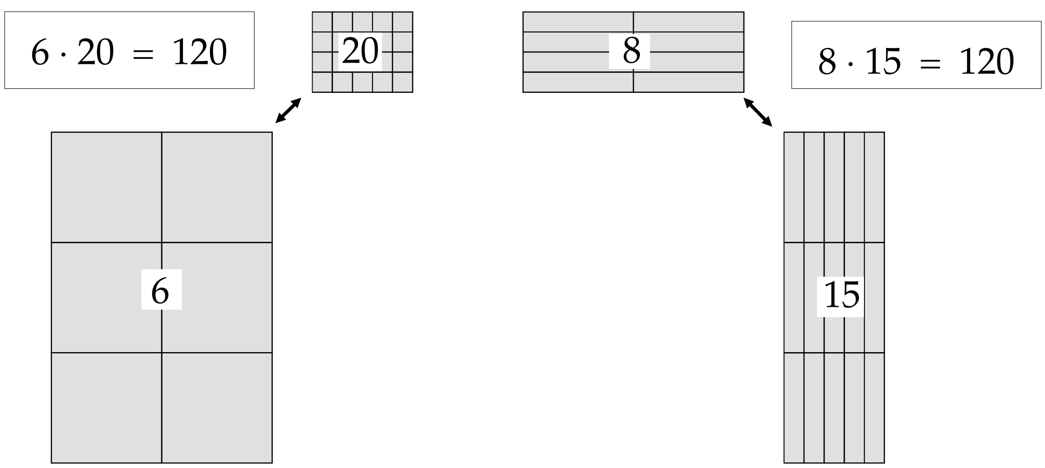
In FACTORING POLYNOMIALS, Section 6, we introduced a shortcut method of factoring. This section is a more detailed explanation of why this method works.

Below is a picture of the rectangle formed when we multiply

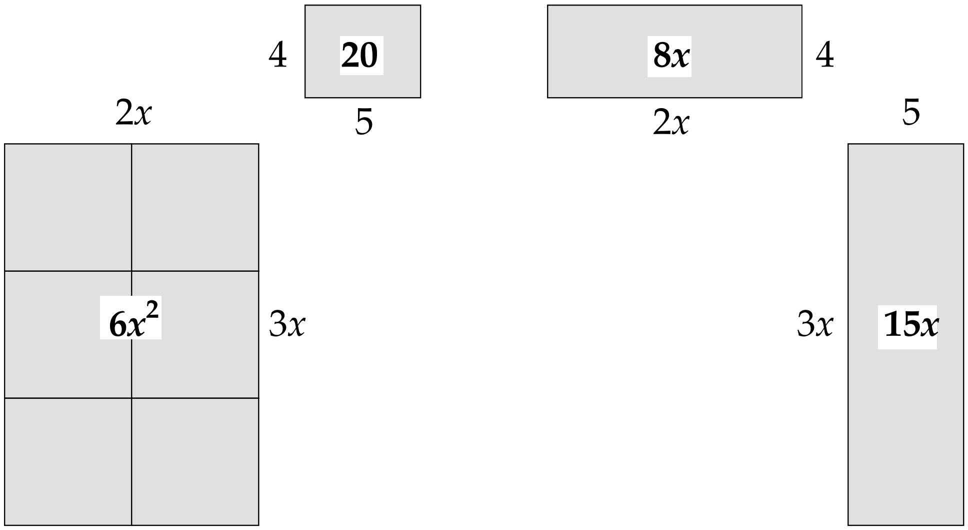
$$(3x+4)(2x+5) = 6x^2 + 15x + 8x + 20$$



You should notice that the number of big squares times the number of small squares (6·20) equals the number of top x-bars times the number of side x-bars (8·15).



If we look at the edges of each rectangle we will understand why these products will always be equal.

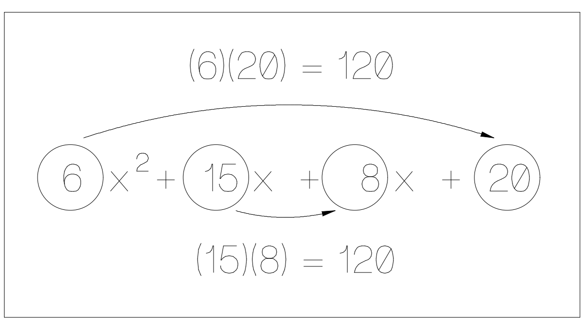


$6 \cdot 20 = (2 \cdot 3) \cdot (4 \cdot 5)$

$8 \cdot 15 = (2 \cdot 4) \cdot (5 \cdot 3)$

Both products are the same because they both have all the same factors, just arranged in a different order.

Looking at the four terms of our product, this means that the number of x-squared pieces (first term) times the number of units (last term) will always equal the product of the numbers of x-bars (middle terms).



Now when the two middle terms are combined, giving

$$6x^2 + 23x + 20$$

we can see that the numbers which added to give us 23 must also multiply to give 120.

Starting with the combined form and working backwards to factor, we can use the method described in the chapter text to break the middle term into its two parts.

Then we can take the common factor from the first two terms and from the last two terms; this results in an amount in parentheses which is the same in both cases. This common factor is one of the factors of the original expression; the other factor is the sum of the pieces multiplying this common factor.

$$6x^2 + 23x + 20$$

$$6x^2 + (15x + 8x) + 20$$

$$(6x^2 + 15x) + (8x + 20)$$

$$3x(2x + 5) + 4(2x + 5)$$

$$(3x + 4)(2x + 5)$$

(6)(20) = 120	
120	1
60	2
40	3
30	4
24	5
20	6
15	**8**
12	10

This method is called **factoring by grouping**; it works for factoring expressions having any number of *x*-squared pieces.