



Manipulative Kits

Basic Kit

The basic manipulative kit for Flip-Chip Algebra has four different types of pieces; these can be used to illustrate all of the topics covered in the two texts *Flip-Chip Algebra* and *Flip-Chip Essentials*. The Basic Kit contains:

□ **Unit Squares:**

The unit squares normally represent single units and are used in multiples to represent the integers. These pieces represent the positive integers when turned colored side up; they represent negative integers when turned with their white side up.

Addition of the integers is shown by sliding together groups of the chips and letting chips of different colors cancel out one for one.

To illustrate multiplication of two integers, make a rectangle with the chips; each of the integers being multiplied is used as the length of one dimension of the rectangle. Beginning with the rectangle turned colored side up, all the chips in the rectangle are flipped once for each negative sign used in the multiplication, with a double flip (two negatives) returning the chips to colored (positive) side up.

Unknown numbers are represented by stacks of chips. Multiples of unknowns are made (without counting) by making several stacks of chips which are all the same height. Positive groups of unknowns are represented by stacks turned colored side up; negative unknowns are represented by stacks of equal height turned white side up.

□ **x rectangles:**

Unknowns and variables can also be represented using x -bars; these bars are like stacks of chips which have been laid out end to end. The x -bars are the same width as the unit squares, but since they represent an unknown number of unit chips, the x -bars are not the

same length as any integral number of unit chips. The x -bars represent stacks of chips which are of unknown height, and therefore should not be replaceable by any specific number of unit chips. As with the unit chips, x -bars turned colored side up represent $+x$, while x -bars turned white side up represent $-x$.

To demonstrate polynomials, add x 's by combining all of the x -bars together and letting those of different colors cancel each other out one for one. Multiplying with x 's is represented by making rectangles where the length of one dimension includes the length of one or more x -bars.

□ **x^2 squares:**

The large squares have both their length and width equal to the length of the x -bars. This means that the area of each large square is $x \cdot x$ or x^2 . As with the other pieces, these large squares turned colored side up represent $+x^2$, and turned white side up represent $-x^2$.

To add polynomials, add the x^2 terms by combining the large squares into one group. The large x^2 squares are used to represent $x \cdot x$ in rectangular arrays (products) which have x 's in the lengths of both dimensions.

□ **y rectangles:**

The shorter bars represent a second unknown (y) having value unequal to that of the first unknown (x). The fact that one bar is shorter than the other merely suggests that the values of x and y are generally different, and does not indicate that y is generally smaller (or larger) than x .

Contents:

- 60 unit squares
- 18 x rectangles
- 6 x^2 squares
- 6 y rectangles

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Extension Kit

□ xy rectangles:

These rectangles have one dimension the same length as the x -bar and the other dimension the same length as the y -bar; they represent the area xy . Colored side up they represent $+xy$; turned white side up these rectangles represent $-xy$.

These pieces are used in polynomials involving both x and y , and in making rectangles representing products involving both x and y .

□ y^2 squares:

The squares which are the same length and width as the y -bars represent $y \cdot y$, or y^2 . Again the colored side represents $+y^2$, and the white side represents $-y^2$. These pieces are used in polynomials involving y , or x and y .

The y^2 squares have been designed to have (approximately) the same area as the x -bars. This feature allows polynomials of the third and fourth order to be represented accurately by the two-dimensional pieces used in the Basic and Extension Kits. If the x -bar is

equal to y^2 , then the xy rectangle $= y \cdot y^2 = y^3$, and the x^2 square $= y^2 \cdot y^2 = y^4$.

□ xy right triangles and $(x - y)^2$ square:

These pieces together can be used to show a simple proof of the Pythagorean Theorem. Lay the longer leg of each triangle along one edge of the red square with the right-angle corners touching, and with the smaller angle (the point) of each triangle pointing clockwise around the red square. You will notice that a larger square is created on the outside of the triangles with each edge of the larger square being the hypotenuse of one of the triangles.

Thus the four triangles and the red square form the square of the hypotenuse. These same pieces can be rearranged and shown to exactly cover the x^2 square plus the y^2 square, proving the theorem.

Contents:

- 6 xy rectangles
- 6 y^2 squares
- 1 $(x - y)^2$ square
- 4 xy right triangles