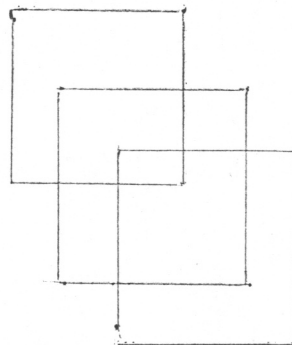
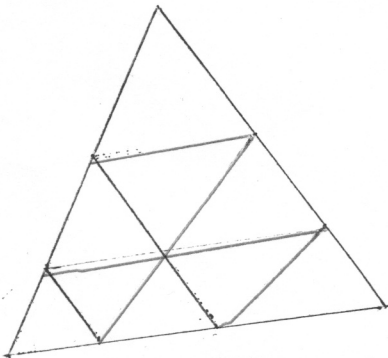
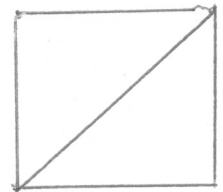
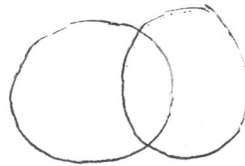
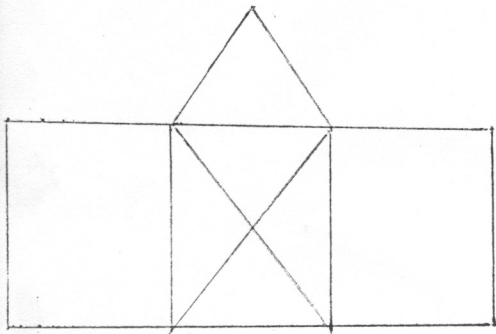
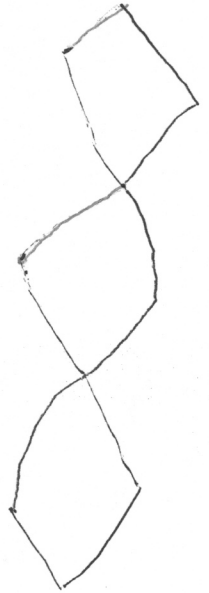
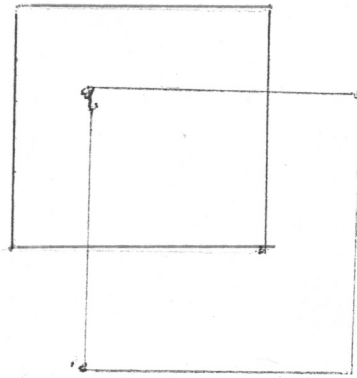
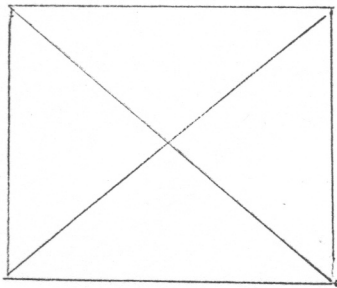
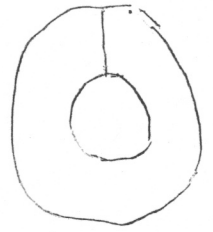
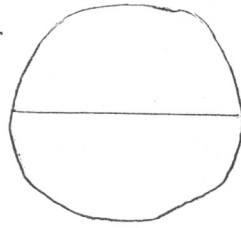
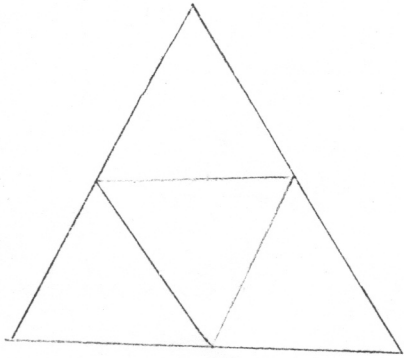


Trace these shapes completely without lifting your pencil and without going over a line twice. You can cross lines through a point.  
It helps to draw on a transparency with an erasable marker.  
Some may not be possible. Try making up your own.



# Discussion

## Drawing Tips:

1. It is much easier to use a pencil or work on a transparency with a dry-erase pen. It is difficult to just draw on the pattern.
2. Advanced students may be able to draw the figure away from the pattern.
3. If possible, mark starting and ending points and show the path taken.

## Learning goals:

1. Notice that some paths have separate starting and ending points, and some paths are closed loops that can start at any point and will end at the starting point. Some problems are not possible.
2. Is it possible to tell which figures can be done and which are impossible?
3. Mark each point in the diagrams with the number of lines coming out from a point (a line that goes straight through a point is counted as two lines coming out).
4. Can you see a pattern of these numbers when looking at the three types: loop, open path with two ends, not possible?

## Answer:

When looking at the numbers on each figure, count the number of points that are even and the number that are odd.

- Closed loop: all even
- Open path: two odd
- Not possible: more than two odd

## Explanation:

When you draw the path, you go into each point and then out, using up two lines. Each time you come to a point, you use up two more lines, so you always use up an even number of points. Odd points are either starting or ending points of the line, and you can only have two points because a line only has two ends. When there are all even point, you can start at any point, but that becomes an odd point so you will end up there.

Closed loops are called Euler Paths, after the Swiss mathematician Leonhard Euler who invented this subject. Open lines are called Semi-Eulerian Paths.

## For further thought:

1. How come you can only have even numbers of odd points?
2. (Advanced) The explanation above for closed loops really only explains that if the loop works, all the points will be even. Why can't there be situations of all even points where the path returns to the starting point but you can't do the whole figure?