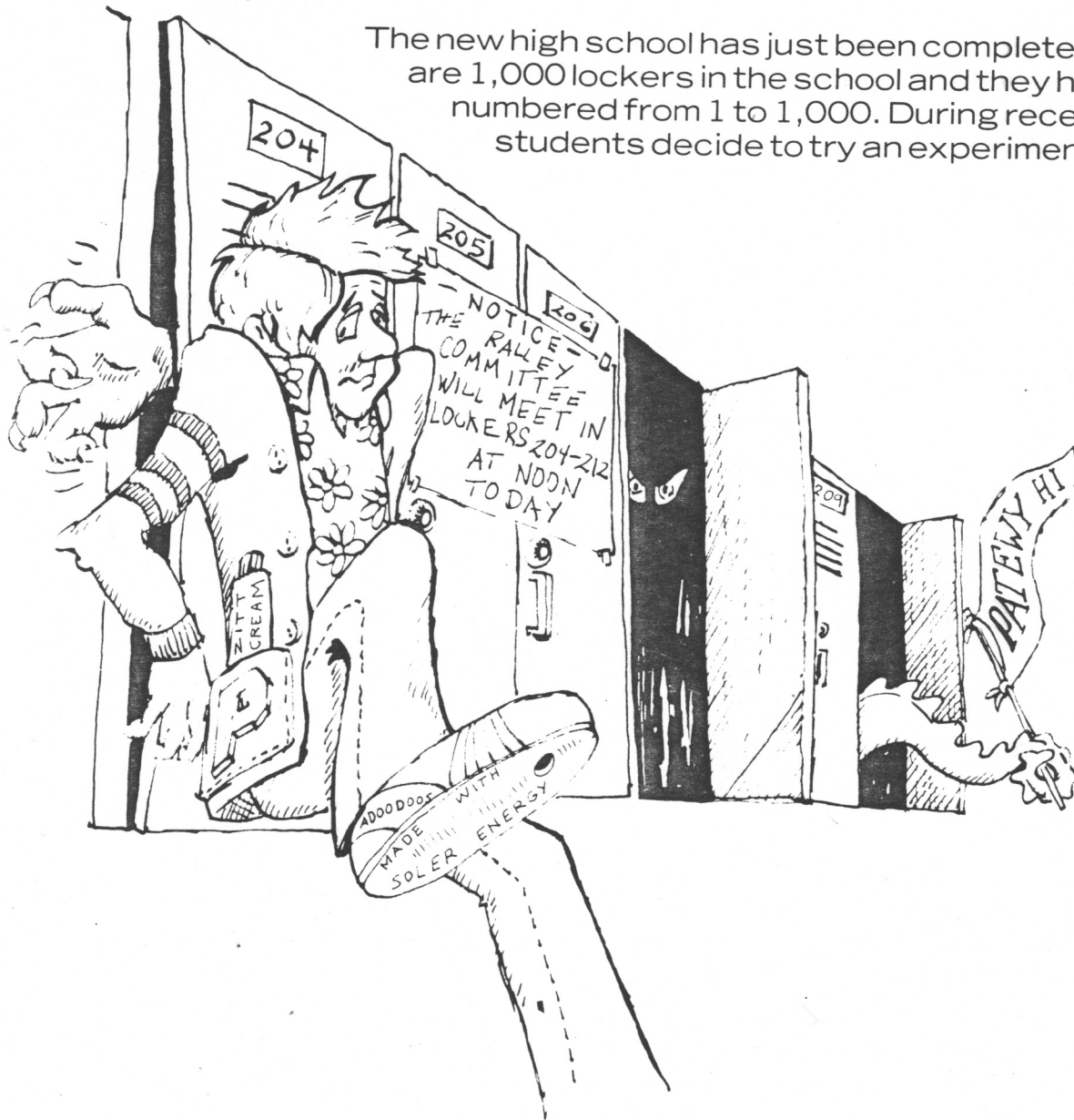# AN OPEN AND SHUT CASE

The new high school has just been completed. There are 1,000 lockers in the school and they have been numbered from 1 to 1,000. During recess, the students decide to try an experiment.



When recess is over, each student will walk into the school one at a time. The first student will open all of the locker doors. The second student will close all of the locker doors with even numbers. The third student will change all the locker doors with numbers that are multiples of three. (*Change* means closing lockers that are open and opening lockers that are closed.) The fourth student will change the position of all locker doors numbered with multiples of four; the fifth student will change the position of the lockers that are multiples of five, and so on. After 1,000 students have entered the school, which locker doors will be open?

| LOCKERS | Start | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 | Visitors | No of Visitors |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | C | O | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | 1 | 1 |
| 2 | C | O | C | | | | | | | | | | | | | | | | | | | | | | | | | | | | | 1, 2 | 2 |
| 3 | C | O | | C | | | | | | | | | | | | | | | | | | | | | | | | | | | | 1, 3 | 2 |
| 4 | C | O | C | | | | | | | | | | | | | | | | | | | | | | | | | | | | | 1, 2, | 3 |
| 5 | C | O | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 6 | C | O | C | O | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 7 | C | O | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 8 | C | O | C | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 9 | C | O | | C | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 10 | C | O | C | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 11 | C | O | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 12 | C | O | C | O | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 13 | C | O | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 14 | C | O | C | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 15 | C | O | | C | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 16 | C | O | C | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 17 | C | O | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 18 | C | O | C | O | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 19 | C | O | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 20 | C | O | C | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 21 | C | O | | C | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 22 | C | O | C | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 23 | C | O | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 24 | C | O | C | O | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 25 | C | O | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 26 | C | O | C | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 27 | C | O | | C | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 28 | C | O | C | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 29 | C | O | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 30 | C | O | C | O | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |

| LOCKERS | Start | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 | Final | Visitors | No of Visitors |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | C | O | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | O | 1 | 1 |
| 2 | C | O | C | | | | | | | | | | | | | | | | | | | | | | | | | | | | | C | 1, 2 | 2 |
| 3 | C | O | | C | | | | | | | | | | | | | | | | | | | | | | | | | | | | C | 1, 3 | 2 |
| 4 | C | O | C | | O | | | | | | | | | | | | | | | | | | | | | | | | | | | O | 1, 2, 4 | 3 |
| 5 | C | O | | | | C | | | | | | | | | | | | | | | | | | | | | | | | | | C | 1, 5 | 2 |
| 6 | C | O | C | O | | | C | | | | | | | | | | | | | | | | | | | | | | | | | C | 1, 2, 3, 6 | 4 |
| 7 | C | O | | | | | | C | | | | | | | | | | | | | | | | | | | | | | | | C | 1, 7 | 2 |
| 8 | C | O | C | | C | | | | O | | | | | | | | | | | | | | | | | | | | | | | C | 1, 2, 4, 8 | 4 |
| 9 | C | O | | C | | | | | | O | | | | | | | | | | | | | | | | | | | | | | O | 1, 3, 9 | 3 |
| 10 | C | O | C | | | O | | | | | C | | | | | | | | | | | | | | | | | | | | | C | 1, 2, 5, 10 | 4 |
| 11 | C | O | | | | | | | | | | C | | | | | | | | | | | | | | | | | | | | C | 1, 11 | 2 |
| 12 | C | O | C | O | C | | O | | | | | | C | | | | | | | | | | | | | | | | | | | C | 1, 2, 3, 4, 6, 12 | 6 |
| 13 | C | O | | | | | | | | | | | | C | | | | | | | | | | | | | | | | | | C | 1, 13 | 2 |
| 14 | C | O | C | | | | O | | | | | | | | C | | | | | | | | | | | | | | | | | C | 1, 2, 7 14 | 4 |
| 15 | C | O | | C | | O | | | | | | | | | | C | | | | | | | | | | | | | | | | C | 1, 3, 5, 15 | 4 |
| 16 | C | O | C | | O | | | | C | | | | | | | | O | | | | | | | | | | | | | | | O | 1, 2, 4, 8, 16 | 5 |
| 17 | C | O | | | | | | | | | | | | | | | | C | | | | | | | | | | | | | | C | 1, 17 | 2 |
| 18 | C | O | C | O | | | C | | | O | | | | | | | | | C | | | | | | | | | | | | | C | 1, 2, 3, 6, 9, 18 | 6 |
| 19 | C | O | | | | | | | | | | | | | | | | | | C | | | | | | | | | | | | C | 1, 19 | 2 |
| 20 | C | O | C | | O | C | | | | | O | | | | | | | | | | C | | | | | | | | | | | C | 1, 2, 4, 5, 10, 20 | 6 |
| 21 | C | O | | C | | | O | | | | | | | | | | | | | | | C | | | | | | | | | | C | 1, 3, 7, 21 | 4 |
| 22 | C | O | C | | | | | | | | | O | | | | | | | | | | | C | | | | | | | | | C | 1, 2, 11, 22 | 4 |
| 23 | C | O | | | | | | | | | | | | | | | | | | | | | | C | | | | | | | | C | 1, 23 | 2 |
| 24 | C | O | C | O | C | | O | | C | | | | O | | | | | | | | | | | | C | | | | | | | C | 1, 2, 3, 4, 6, 8, 12, 24 | 8 |
| 25 | C | O | | | C | | | | | | | | | | | | | | | | | | | | | O | | | | | | O | 1, 5, 25 | 3 |
| 26 | C | O | C | | | | | | | | | | | C | | | | | | | | | | | | | C | | | | | C | 1, 2, 13, 26 | 4 |
| 27 | C | O | | C | | | | O | | | | | | | | | | | | | | | | | | | | C | | | | C | 1, 3, 9, 27 | 4 |
| 28 | C | O | C | | O | | | C | | | | | | | O | | | | | | | | | | | | | | C | | | C | 1, 2, 4, 7, 14, 28 | 6 |
| 29 | C | O | | | | | | | | | | | | | | | | | | | | | | | | | | | | C | | C | 1, 29 | 2 |
| 30 | C | O | C | O | | C | O | | | | C | | | | | O | | | | | | | | | | | | | | | C | C | 1, 2, 3, 5, 6, 10, 15, 30 | 8 |