

Card Trick - Pairs

Setup:

Take 4 playing cards, 2 red and 2 black (only colors matter) Shuffle the cards face down and draw 2.

Question:

What is the probability of getting two cards that are the same color?

Discussion:

What are possible answers and justifications for those answers. The answer is not obvious ($\frac{1}{2}$?, $\frac{1}{4}$? $\frac{2}{3}$ $\frac{1}{3}$?) and many answers will seem reasonable. How can we settle the matter? Do it multiple times and see how often the pair drawn is two of the same color (In a class, have everyone do it ten or more times and combine results).

Answers:

The experiment should show approximately one-third. There are at least three ways to explain this:

1. Pairs in order – There are 4 possibilities for the first card, 3 for the second card, so $4 \times 3 = 12$ possible draws. R1 R2, R2 R1, B1 B2, B2 B1 or $4/12 = \frac{1}{3}$ is the probability.
2. Pairs, ignore order – There are $4 \times 3 = 12$ pairs, but look at 6 ignoring the order. There are 2 same-color pairs, so 2 out of 6 = $2/6 = \frac{1}{3}$.
3. A quick answer - After you draw the first card, you have 3 cards left, with only one being a match, so 1 out of 3 = $\frac{1}{3}$.

This problem is a good example of a question where just reasoning gives several possible answers ($\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{3}$), so you can use inductive reasoning to do an experiment, find the answer, and then find the reasoning. The resulting “proof” comes after all of the experimenting.

Especially with problems of probability, it is a good idea to do a problem in more than one way to make sure you have it right.

[From Innumeracy, by John Dos Paulos]