

Three - Door Game Show

Setup:

You are on a game show where there are three closed doors; you are told that one door has a million-dollar prize and the other two are empty. You are asked to choose one of the three doors; before it is opened, the host opens one of the other two doors and shows you it is empty. You now have the choice to stick to your original door or switch to the other one.

Question:

What should you do – stay or switch? Or it doesn't matter and they both equally likely to be winners?

Discussion:

This question can be answered reasonably with either choice. The arguments can be persuasive for both sides, so how do you know what is right? Individually or in a class, you can actually play the game a number of times and see what happens. You can count how many times you win with each strategy, or assign a team to switch and a team to stay, and see which does best.

Answer:

You should switch. You are more likely to win if you switch, and it does matter. The right answer involves looking at the probabilities from the start and noticing that the host's action showing you one of the empty doors has given you more information. Let's do it in two ways:

1. When you first pick a door, you have a $\frac{1}{3}$ chance of winning, and if you stay, that is still your chance of success, so switching has a probability of $1 \text{ minus } \frac{1}{3} = \frac{2}{3}$.
2. If you look at the three possibilities (doors you chose from), when the host looks at the two other doors and shows you one, in two cases one of the other doors has the prize, and in one case the prize is in your door, so staying wins in 1 case and switching wins in 2 cases. Out of 3 cases, this is $\frac{1}{3}$ and $\frac{2}{3}$.

It is always best to have several ways that agree if you want to be sure.